Pergamon

www.elsevier.com/locate/actaastro

PII: S0094-5765(02)00113-3

Acta Astronautica Vol. 51, No. 12, pp. 873–878, 2002

Printed in Great Britain
0094-5765/02/$ - see front matter

MAKING A TUNNEL THROUGH THE MOON

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(Received 7 June 2000; revised version received 26 April 2001)

Abstract—The pressure in the center of the moon is estimated to be 50,000 atm at a temperature of a few 10^3 K. Under these conditions a tunnel to the center of the moon seems possible, if dug by a sequence of nuclear explosions, crushing the rocks through which the tunnel shall pass. The crushed rocks reduce the pressure gradient in the tunnel wall and permit the removal of heat by liquid metals. The number of required nuclear explosions, estimated to be several thousand, can be substantially reduced by thermonuclear shape charges. © 2002 Elsevier Science Ltd. All rights reserved

1. INTRODUCTION

The idea of making a tunnel to the center of the moon with a chain of nuclear explosives was suggested many years ago by the author in a semipopular magazine [1] In this article, I did not give a mathematical analysis of this concept which, at the end of a rich and long scientific career, I now find the time to supply.

The same concept, of course, could as well be used to make a deep mine, perhaps as deep as ≈ 40 km. Quite apart from its scientific importance, such a tunnel to the center of the moon could have great economical benefits. It is generally believed that heavy metals are concentrated in the center of planetary bodies, where they are accumulated during the liquid formative phase.‡

The idea to make a vertical tunnel into the moon is shown schematically in Fig. 1. At the head of the tunnel, a nuclear explosion is set off, shattering the surrounding rocks relaxing the pressure gradient through the buildup of large shear stresses. The tunnel is then dug through the crushed rocks, with the heat removed by liquid metals passing through the pores in the crushed rock.

How deep the tunnel can be made depends on the pressure rocks can withstand, which is below 100,000 atm. Since the pressure in the center of the moon is less, a tunnel to the center of the moon seems possible.

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‡Especially the planet Mercury, with its high specific density, should be rich in heavy metals.

2. THE PRESSURE AND TEMPERATURE IN THE CENTER OF THE MOON

The radius of the moon is $R = 1.74 \times 10^8$ cm, and the gravitational acceleration at its surface $g_0 \cong 1.62 \times 10^2$ cm/s^2. At a distance $r < R$ from its center the gravitational acceleration is

$$g(r) = -g_0(r/R) \tag{1}$$

and the pressure balance equation is

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\rho g_0 \frac{r}{R}, \tag{2}$$

where $p = p(r)$ is the pressure for $r < R$. The pressure at the center therefore is

$$p_{\max} = \frac{\rho g_0}{R}\int_0^R r\,\mathrm{d}r = \frac{1}{2}\rho g_0 R. \tag{3}$$

With $\rho \cong 3.33$ g/cm^3 the average density of the moon, one finds that $p_{\max} \cong 5 \times 10^{10}$ dyn/cm^3 ≅ 50,000 atm.

The temperature T can be estimated from the equation $p_{\max} = nkT$, where $k = 1.38 \times 10^{-16}$ erg/K is the Boltzmann constant and $n \cong 10^{23}$ cm^{-3} the atomic number density of the rocks. For $p_{\max} = 5 \times 10^{10}$ dyn/cm^2 one finds $T \cong 4 \times 10^3$ K.

3. SHATTERING OF THE LUNAR ROCKS BY NUCLEAR EXPLOSIONS

The cohesive energy of the rocks is of the order $\varepsilon_r \approx 10^{10}$ erg/cm^3. Therefore, the explosive yield needed to shatter a spherical volume of radius r is

$$E \cong (4\pi/3)\varepsilon_r r^3. \tag{4}$$

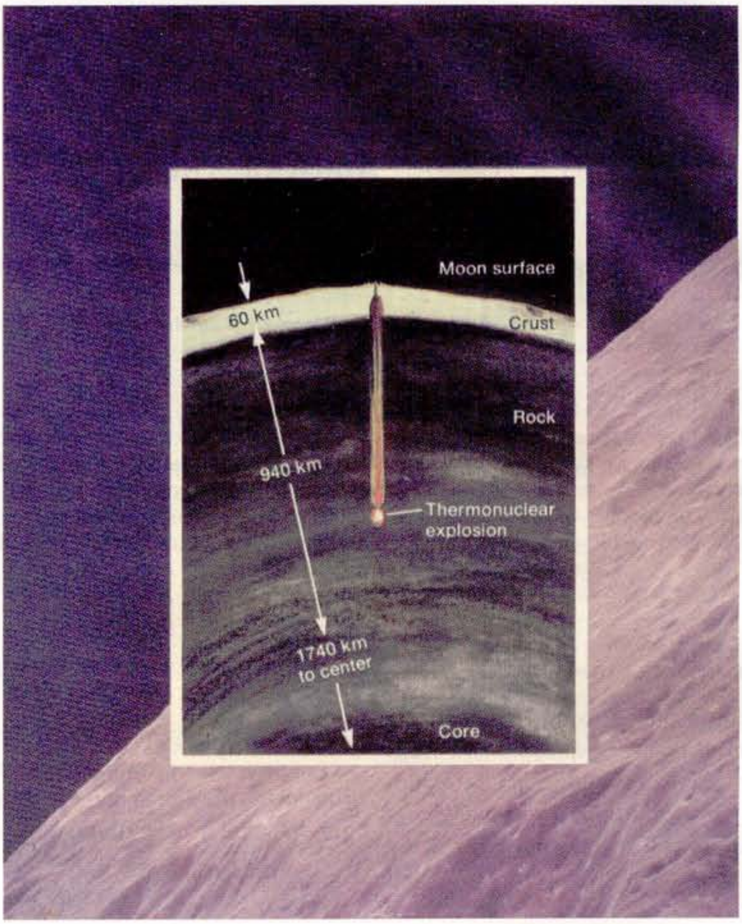

Fig. 1. Making a tunnel through the Moon.

The energy released in a kiloton nuclear explosion is $E = 4 \times 10^{19}$ erg. With this energy, the radius of the crushed rocks would be $r \cong 10^3$ cm (=10 m), and with a 10 kt explosion it would be twice as large.

To make a tunnel, a cylindrical rather than spherical volume of crushed rocks is desired. For this reason a thermonuclear shape charge or an explosive lens is better suited to shatter the rocks. This possibility will be discussed below.

4. REMOVAL OF HEAT FROM THE CRUSHED HOT ROCKS

In the center of the moon the temperature is several thousand degrees centigrade. With the heat diffusion equation given by

$$\frac{\partial T}{\partial t} = \chi \nabla^2 T, \tag{5}$$

where χ is the heat diffusion coefficient, the diffusion time for a layer of thickness x is

$$\tau = x^2/\chi. \tag{6}$$

For lunar rocks one has $\chi \cong 4 \times 10^{-3}$ cm^2/s. Taking the example x=20 m=2×10^3 cm, one finds that τ= 10^9 s $\cong$ 30 yr, and at the high rock temperatures the heat diffusion time would be uncomfortably long.

The situation is drastically changed for a layer of crushed rocks, because there it is possible to remove the heat by a coolant pumped through the porous medium formed by the crushed rocks. At the high temperatures of several thousand degrees centigrade, a liquid alkali metal, for example lithium, abundantly available on the moon, could be used as a coolant. The velocity the coolant diffuses into the crushed rocks is determined by Darcy's law

$$v = -D \,\mathrm{grad}\, p \tag{7}$$

where p is the pressure, $D=\kappa/\rho g$, with $\kappa \sim 1$ cm/s a typical value. If the pressure gradient is provided by the gravitational force one has grad $p = \rho g$ and hence

$$|v| = \kappa \sim 1 \text{ cm/s}. \tag{8}$$

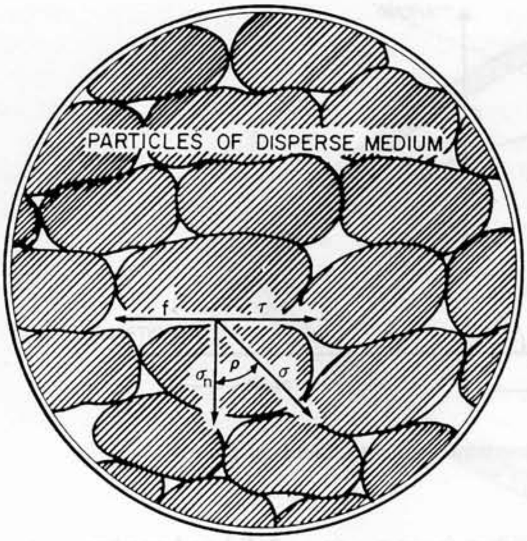

Fig. 2. The sustaining of large shear stresses in a disperse medium by frictional forces between the particles of the medium.

The time needed for the liquid metal to pass through a $\sim$ 20 m thick layer is then $\sim 2 \times 10^3$ s $\sim$ 1 h. The specific heat per unit volume of the coolant is $\rho c_v \sim 3 \times 10^7$ erg/cm^3 K, and for $T = 3 \times 10^3$ K, one has $\rho c_v T \sim 10^{11}$ erg/cm^3.

The heat per unit volume which has to be removed from the crushed rocks is of the order p, where p is the rock pressure. In the center of the moon where $p = 5 \times 10^{10}$ dyn/cm^2, this energy is 5×10^{10} erg/cm^3. It thus follows that the volume of the liquid coolant must be about 1/2 of the rock volume to be cooled. For a rock volume of $(20\ \mathrm{m})^3 \approx 10^4\ \mathrm{m}^3$, a coolant volume of about 5×10^3 m^3 would be needed. The same coolant can be used many times over after the heat is removed from it, which could be done on the surface of the moon by radiation, or perhaps better by heat exchangers transferring the heat to lunar sand.

5. PRESSURE DISTRIBUTION IN THE SHATTERED ROCKS FOLLOWING A NUCLEAR EXPLOSION

Without a thick layer of shattered rocks surrounding the tunnel, the pressure acting on the tunnel wall would be very large, in particular, in the center of the moon. Because of friction between the particles of the shattered rock, large shear stresses can be sustained changing the pressure distribution in the rock reducing the pressure gradient and hence the pressure on the tunnel wall.

This effect is shown in Fig. 2. If σ is the compressive stress acting in some direction with regard to the surface of a crushed rock particle, a shear stress τ parallel to its surface will be set up. This shear stress is compensated by a friction force acting in equal and opposite direction of the shear as long as

$$\tau \leqslant \sigma_n \tan \rho, \tag{9}$$

where ρ is the friction angle and σ_n the normal component of the compressive stress σ. If plotted in the Mohr stress diagram, the maximum possible shear cannot exceed the boundary line $\tau = \sigma \tan \rho$, as shown in Fig. 3. For a compressive load σ is negative and the boundary line $\tau = \sigma \tan \rho$ has a negative slope. The maximum possible shear is

$$\tau_{\max} = (\sigma_{\max} - \sigma_{\min})/2 \tag{10}$$

in the Mohr diagram given by

$$\sin \rho = \frac{\sigma_{\max} - \sigma_{\min}}{\sigma_{\max} + \sigma_{\min}}. \tag{11}$$

Hence

$$\sigma_{\min}/\sigma_{\max} = \tan^2(45^\circ - \rho/2) \tag{12}$$

with this function plotted in Fig. 4. A typical value for the friction angle is $\rho \approx 45^\circ$, resulting in $\sigma_{\min}/\sigma_{\max} \cong 0.1$.

With σ_{ik} the stress tensor in the medium of the crushed rocks, the static equilibrium equation is in Cartesian coordinates given by

$$\frac{\partial \sigma_{ik}}{\partial x_k} = 0. \tag{13}$$

In the posed problem, we assume cylindrical and spherical symmetry, the latter for the geometry at the lower end of the tunnel shaft. Introducing curvilinear coordinates the static equilibrium eqn (13) becomes

$$\sigma^k_{i;k} = 0, \tag{14}$$

where the colon stands for the covariant derivative. For eqn (14) one can write

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^k}(\sqrt{g}\sigma^k_i) - \Gamma^\ell_{ik}\sigma^k_\ell = 0 \tag{15}$$

with the square of the line element $\mathrm{d}s^2 = g_{ik}\,\mathrm{d}x^i\,\mathrm{d}x^k$ defining the metric tensor and $g = \det g_{ik}$. The Γ^ℓ_{ik} are the Christoffel three index symbols of the second kind.

In the cylindrical case, assuming $\partial/\partial\phi = 0$, we have for a cylindrical r, ϕ, z coordinate system $\Gamma^1_{11} = 0$, $\Gamma^2_{21} = 1/r$, and $\sqrt{g} = r$. Here we find from eqn (14)

$$r\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} + \sigma_r - \sigma_\phi = 0, \tag{16}$$

where $\sigma^1_1 \equiv \sigma_r$, $\sigma^2_2 \equiv \sigma_\phi$, $\sigma^3_3 = 0$, with σ_r and σ_ϕ the stress components in the r and ϕ directions.

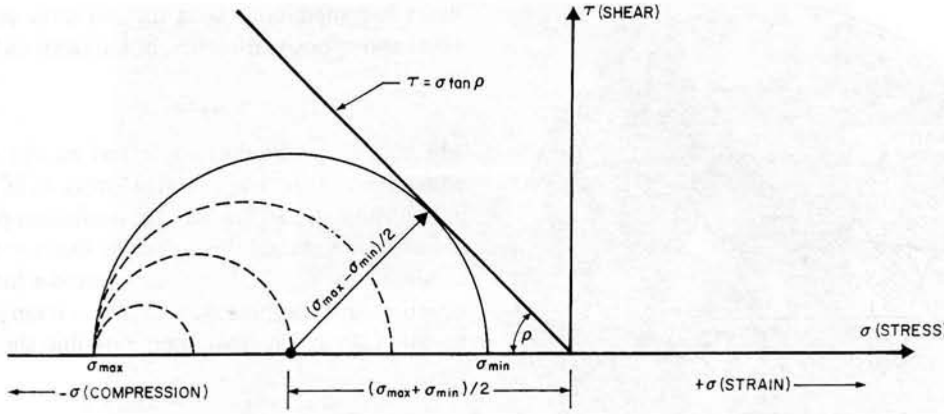

Fig. 3. Mohr stress diagram for disperse medium subject to fractional forces.

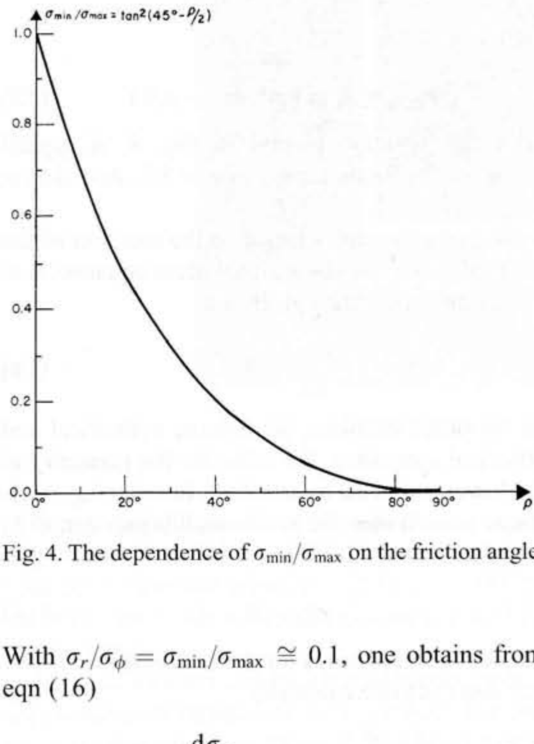

Fig. 4. The dependence of $\sigma_{\min}/\sigma_{\max}$ on the friction angle.

With $\sigma_r/\sigma_\phi = \sigma_{\min}/\sigma_{\max} \cong 0.1$, one obtains from eqn (16)

$$r\,\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} - 9\sigma_r = 0. \tag{17}$$

Hence (putting $\sigma_r(r) = p(r)$) one obtains by integrating (17)

$$p = p_0(r/r_0)^9, \tag{18}$$

where p_0 is the pressure at the tunnel wall and r_0 its radius.

In spherical coordinates of a, r, θ, ϕ spherical polar coordinate system one has $\Gamma^1_{11} = 0$, $\Gamma^2_{12} = \Gamma^3_{13} = 1/r$, and $\sqrt{g} = r^2 \sin\theta$. Here one finds that

$$r\,\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} - 2(\sigma_r - \sigma_\theta) = 0 \tag{19}$$

with $\sigma_r/\sigma_\theta = \sigma_{\min}/\sigma_{\max} \cong 0.1$ and one has

$$r\,\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} - 18\sigma_r = 0 \tag{20}$$

with the result that

$$p = p_0(r/r_0)^{18} \tag{21}$$

for the pressure distribution in the semispherical cap of crushed rocks at the head of the tunnel.

Assuming that the pressure inside the tunnel with radius r_0 at the center of the moon is of the order 1 atm, but at a radius $r > r_0$ equal to the 50,000 atm, the radius of the crushed rock layer must be related to the tunnel radius r_0 by $r/r_0 \cong (5\times10^4)^{1/9} \cong 3.3$. Hence, for an assumed tunnel radius of 20 m in the center of the moon, the layer of crushed rocks would have to be 70 m thick.

The radius of the semispherical cap at the head of the tunnel, computed from $r/r_0 = (5\times10^4)^{1/18} \cong 2$, is $r \sim 40$ m.

Farther away from the center of the moon the pressure is smaller and with it the required layer of crushed rocks, which in turn requires smaller nuclear explosions to crush the rocks.

6. THE NUMBER OF NUCLEAR EXPLOSIONS REQUIRED TO MAKE THE TUNNEL

Integrating eqn (2) one obtains for the pressure distribution in the moon

$$p(r) = -\frac{\rho g_0}{R}\int_R^r r\,\mathrm{d}r = \frac{\rho g_0}{2R}(R^2 - r^2) \tag{22}$$

for which one can also write

$$p(r) = p_{\max}\left(1 - \left(\frac{r}{R}\right)^2\right). \tag{23}$$

If r_s is the horizontal radius up to which the rocks at a certain depth have to be shattered (cylindrical

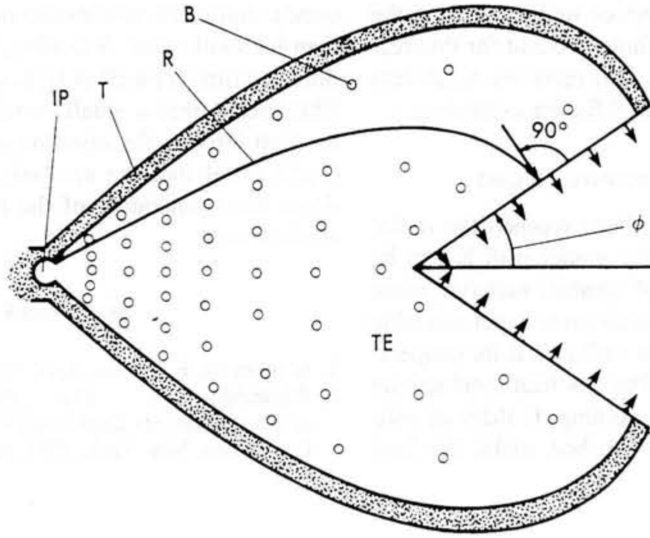

Fig. 5. A wave-shaping lens for conical implosion where the detonation lens produces a convergent conical wave. IP is the ignition point of the thermonuclear explosive TE; B are the bubbles placed in the wave path; T is the tamp; and R is a ray of the detonation wave.

case), one finds by equating $p(r)$ in (18) and (23)

$$r_s = r_0(p_{max}/p_0)^{1/9}(1-(r/R)^2)^{1/9}. \quad (24)$$

The total energy required to shatter the rocks to make a tunnel from the center at the moon where $r = 0$, to its surface where $r = R$, is then given by summing up over the slices with radius r_s and thickness dr

$$E = \pi r_0^2 \varepsilon_r (p_{max}/p_0)^{2/9} \int_0^R (1-(r/R)^2)^{2/9}\, dr, \quad (25)$$

where as in eqn (4) $\varepsilon_r \cong 10^{10}$ erg/cm^3 is the cohesive binding energy of the rocks. For eqn (25) one can write

$$E = \pi r_0^2 R \varepsilon_r (p_{max}/p_0)^{2/9} \int_0^1 (1-x^2)^{2/9}\, dx. \quad (26)$$

With the help of Euler's betafunction one has

$$\int_0^1 (1-x^2)^{2/9}\, dx = (1/2)B\left(\frac{1}{2}, \frac{11}{9}\right) \cong \sqrt{\pi}/2. \quad (27)$$

Hence

$$E \cong (\pi^{3/2}/2) r_0^2 R \varepsilon_r (p_{max}/p_0)^{2/9}. \quad (28)$$

Inserting $r_0 = 2 \times 10^3$ cm, $R = 1.74 \times 10^8$ cm, $\varepsilon_r = 10^{10}$ erg/cm^3, $p_{max}/p_0 = 5 \times 10^4$, one finds that $E \cong 2 \times 10^{24}$ erg $\cong 5 \times 10^4$ kt $= 50$ Mt.

It must be emphasized that this energy must be quite nonuniformly released along the tunnel shaft. Nuclear fission explosions below a yield of 10 kt become uneconomical with only a fraction of the energy in the fissionable material (needed to make a critical assembly) released. For a 10 kt fission explosion the shatter radius computed from (4) is ~ 20 m. With a tunnel radius $r_0 \sim 10$ m one would have $r_s/r_0 \sim 2$ and from (24) that

$$1-(r/R) \cong 2^9(p_0/p_{max}). \quad (29)$$

Putting for the depth of the tunnel (if measured from the surface of the moon) $\delta = R - r$, with $\delta/R \ll 1$, one finds from (29) that

$$2\delta/R \cong 2^9(p_0/p_{max}) \cong 10^{-2} \quad (30)$$

or that $\delta \cong 10$ km.

For a depth < 10 km the nuclear explosion with a yield < 10 kt would suffice, a yield which is uneconomical. It is for this reason suggested to use altogether thermonuclear explosive devices where the cost per yield is much lower. To penetrate and shatter the rocks more efficiently, jet-generating thermonuclear explosive lenses could be used. A design for such a lens configuration is shown in Fig. 5 [2]. The thermonuclear detonation wave ignited at one point is there shaped into a jet producing conical implosion by placing obstacles into the path of the wave. The ignition can be done by a fission explosive, but conceivably also by a powerful laser beam, with the laser beam projected down the tunnel shaft, triggering the thermonuclear explosive positioned at the lower end.

With the above-given estimate of $\sim$ 50 Mt needed to dig the tunnel shaft, the number of thermonuclear explosive devices making use of the detonation wave lens technique could for this reason be quite reasonable, and certainly much less than the number of required fission explosives.

7. MAKING THE TUNNEL TO LAST

After nuclear explosions have crushed the rocks, and the heat removed, the tunnel wall has to be made from some kind of ceramic material, since water with which to make concrete is not available on the moon. But for the wall to last its temperature must be kept low. The low heat conductivity of rocks, requiring little cooling, is there of considerable help. For the crushed rocks the heat conduction coefficient should not be very different than for solid rocks. According to eqn (6) the heat diffusion time for a 20 m layer of rocks is $\approx$ 30 yr. This means that a small, continuous removal of the heat through the injection and circulation of a liquid metal into the crushed rocks should keep down the temperature of the tunnel wall and its environment.

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2. Winterberg, F. *The Physical Principles of Thermonuclear Explosive Devices*. Fusion Energy Foundation, New York, 1981, p. 117 ff.