Mini Fission-Fusion-Fission Explosions (Mini-Nukes). A Third Way Towards the Controlled Release of Nuclear Energy by Fission and Fusion

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Chemically ignited nuclear microexplosions with a fissile core, a DT reflector and U238 (Th232) pusher, offer a promising alternative to magnetic and inertial confinement fusion, not only burning DT, but in addition U238 (or Th232), and not depending on a large expensive laser of electric pulse power supply. The prize to be paid is a gram size amount of fissile material for each microexplosion, but which can be recovered by breeding in U238.

In such a "mini-nuke" the chemical high explosive implodes a spherical metallic shell onto a smaller shell, with the smaller shell upon impact becoming the source of intense black body radiation which vaporizes the ablator of a spherical U238 (Th232) pusher, with the pusher accelerated to a velocity of $\sim$ 200 km/s, sufficient to ignite the DT gas placed in between the pusher and fissile core, resulting in a fast fusion neutron supported fission reaction in the core and pusher. Estimates indicate that a few kg of high explosives are sufficient to ignite such a "mini-nuke", with a gain of $\sim 10^3$, releasing an energy equivalent to a few tons of TNT, still manageable for the microexplosion to be confined in a reactor vessel.

A further reduction in the critical mass is possible by replacing the high explosive with fast moving solid projectiles. For light gas gun driven projectiles with a velocity of $\sim$ 10 km/s, the critical mass is estimated to be 0.25 g, and for magnetically accelerated 25 km/s projectiles it is as small as $\sim$ 0.05 g.

With the much larger implosion velocities, reached by laser- or particle beam bombardment of the outer shell, the critical mass can still be much smaller with the fissile core serving as a fast ignitor.

Increasing the implosion velocity decreases the overall radius of the fission-fusion assembly in inverse proportion to this velocity, for the 10 km/s light gas gun driven projectiles from 10 cm to 5 cm, for the 25 km/s magnetically projectiles down to 2 cm, and still more for higher implosion velocities.

Key words: Fusion-Fission; Impact Fusion; Fast Ignition.

1. Introduction

With magnetic and inertial fusion the two principal avenues towards the controlled release of nuclear energy by nuclear fusion, a third way, the fission assisted release of thermonuclear fusion energy has been almost forgotten.

The original idea was to set off a chain of fission-bomb triggered thermonuclear underground explosions, with heat extracted from the hot cavities formed by these explosions. Having come under the name "Pacer", the concept is certainly feasible, but in view of the magnitude of the explosions, but also because of potential radioactive contamination problems, not very attractive.

Then in 1973, I had suggested that by laser- or relativistic electron beam induced compression of a fissile pellet surrounded by a shell of DT, astonishingly small critical masses seem possible [1, 2]. The feasibility of the idea was questioned by Cole and Renken [3], both from the Sandia National Weapons Laboratory, but as I had explained in a reply [4], these Sandia scientists had overlooked the crucial point of my proposal, which was that the DT reflector surrounding the fissile pellet is "active" in the sense that in it thermonuclear reactions can take place. Because if the DT is heated under the compressive force to high temperatures, the fission reaction rate in the fissile core is greatly increased. And the heat released in the fissile core can in turn increase the temperature of the DT reflector and hence its fusion reaction rate.

It was furthermore shown that by imploding the DT with a shell of U238 (or Th232), the fast fission reactions in the shell by the 14 Mev DT fusion reaction, can "autocatalytically" increase the implosion velocity of the shell [5]. By "autocatalytic" is meant that the implosion velocity is increased by the explosion of the shell through the fast neutron fission reactions in the shell, resulting in an increased implosion velocity of the shell, increasing the DT reaction rate, which in turn releases even more fast neutrons into the shell.

The "active", fast neutron producing DT reflector, in combination with the "autocatalytic" implosion process, makes possible to ignite such a fission-fusion-fission assembly even with a modest amount of a chemical high explosive.

I had first used the term "mini-nuke" in an unpublished report following my 1973 paper in Nature [1], but this terminology should not be confused with the recent use of this same word for small nuclear explosive devices intended for military applications[1].

2. Mini-Nuke Configuration and Yield

Figure 1 shows a cross section through a mini-nuke. At its center is the fissile core of uranium 235 or plutonium 239 with the radius r_0. It is surrounded by the DT reflector inside a shell of radius r_1 serving as pusher and confining the DT gas under a pressure of ~ 200 atm, with a DT particle number density equal to $\sim 1/10$ solid density. At its outer surface the pusher is covered with an ablator, which ideally is a layer of beryllium with the pusher made from U238 or Th232. Surrounding the pusher-ablator shell is an aluminum radiator shell of radius r_2, which in turn is surrounded by a larger aluminum shell of outer radius r_3, on its outer side being covered with a several cm thick layer of a high explosive. The space in between the outer concentric shells is vacuum.

Following the simultaneous ignition of the whole outer surface of the high explosive, the aluminum shell of radius r_3 is imploded on the inner aluminum shell of radius r_2 (see Appendix A). Upon impact, the inner aluminum shell becomes the source of intense black body radiation (see Appendix B), vaporizing the beryllium ablator, with the pusher of radius r_1 compressing and heating the DT together with the fissile core resulting in the onset of the fission-fusion chain reaction (see Fig. 2).

[1]A possible military application of the author's mininuke concept was discussed in a secret 1987 report of the former E. German Government, recently made public by the "Bundesbeauftragte für die Unterlagen des Staatssicherheitsdienstes der ehemaligen Deutschen Demokratischen Republik: Zentralarchiv, MfS-AGM 1001".

We present here the following example:

$$r_0 = 0.25 \text{ cm},\ r_1 = 0.5 \text{ cm},\ r_2 = 1.0 \text{ cm},\ r_3 = 10 \text{ cm}.$$

The high explosive accelerates the outer aluminum shell from an initial velocity of ~ 5 km/s at $r = r_3$, to ~ 50 km/s at $r = r_2$, with the pusher at $r = r_1$ accelerated to a velocity of ~ 200 km/s heating the DT to $\sim 2 \times 10^7$ K. The onset of fusion reactions in the DT reflector leads to fission reactions in the pusher by the autocatalytic process (see Appendix C), and by the fission-fusion process (see Appendix D) reducing the critical mass ~ 50 fold. With a final pressure of several 10^{14} dyn/cm^2, reached at an implosion velocity of several 100 km/s, the fissile core is compressed ~ 10 fold, reducing the critical mass 100 fold. The total reduction in the critical mass therefore is $50 \times 100 = 5 \times 10^3$ fold. Assuming a critical mass of ~ 5 kg for uncompressed fissile material, this implies a critical mass of 1 gram with the radius $r_0 = 0.25$ cm for the fissile core. Assuming a 10% fuel burn up, one would have a yield of 10 GJ $\cong$ 2 tons TNT, dissipated into the combustion products of the high explosive.

A ~ 2.5 cm thick layer of high explosive at the radius $r_3 = 10$ cm, would require ~ 10 kg of high explosive with an energy of ~ 40 MJ, implying a gain of the order 250. Still larger gains should be possible with improved energy focusing of the chemically induced implosion process.

Finally, we turn to the question of stability, that is the Raylaigh-Taylor instability. Theoretical and experimental studies made with capsules imploded by the Nova laser showed sufficient stability for convergence ratios $R_{\text{capsule}}/R_{\text{fuel}} \simeq 20$ [6], which has to be compared with the ratios $r_3/r_2 = 10$, $r_2/r_1 = 2$, $r_1/r_0 = 2$, $r_3/r_0 = 40$. Therefore, with the exception of r_3/r_0, all the ratios are within the limit of feasibility against Rayleigh-Taylor instability growth. The ratio $r_3/r_0 = 40$, larger by a factor 2, applies to a three-shell configuration which has a higher stability. There can be no doubt that high explosives can be made sufficiently uniform and homogenous to reach the same implosion symmetry as with lasers.

3. Replacing the High Explosive with Fast Moving Projectiles

A further substantial reduction in the critical mass and radial assembly dimensions is possible if one replaces the high explosive with fast moving projectiles.

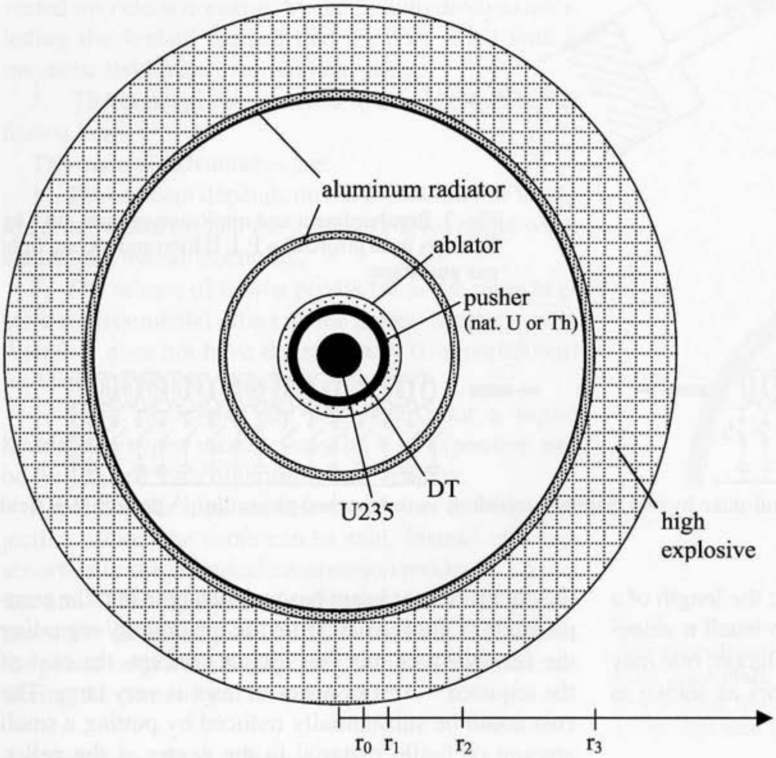

Fig. 1. Mini-Nuke Cross Section.

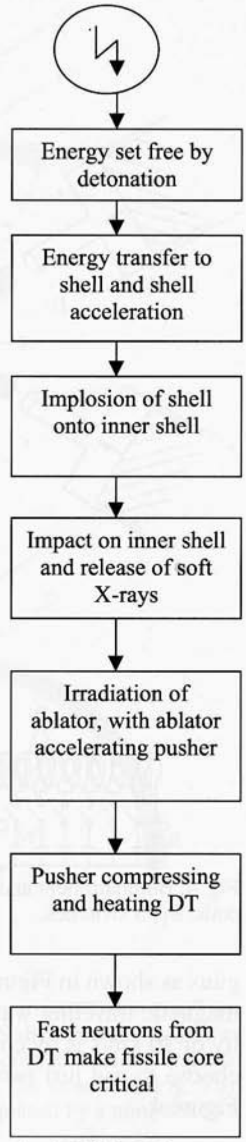

Fig. 2. Mini-Nuke Energy Flow Diagram.

In this way the concept has some similarity with impact fusion [7], except that it works with much smaller velocities. With light gas guns projectile velocities of ~ 10 km/s can be reached, and with magnetic traveling wave accelerators less than 100 m long, velocities which are twice as large [8]. The reduction in the critical mass and the dimension of the mini-nuke are obtained by scaling laws. With the impact pressure p going in proportion to the square of the impact velocity v, and the density of a Fermi gas going in proportion to $p^{3/5}$, the density ρ goes in proportion to $v^{6/5}$. The critical mass is proportional to $1/\rho^2$ and hence proportional to $v^{12/5}$. Therefore, increasing the impact velocity from $v = 5$ km/s to $v = 10$ km/s reduces the critical mass by the factor $2^{12/5} \simeq 5$ from ~ 1 g to 0.2 g, and for $v = 20$ km/s by the factor $4^{12/5} \simeq 27.5$ to 0.04 g. The radial assembly dimension goes down from $r_3 = 10$ cm to $r_3 = 5$ cm, and $r_3 = 2.5$ cm.

By replacing the high explosive with light gas gun driven projectiles one may use a larger number of these

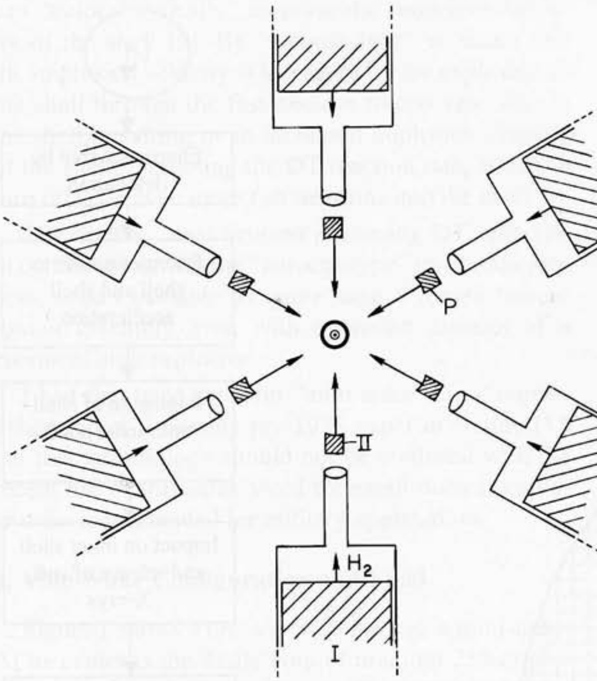

Fig. 3. Bombardment and implosion of mini-nuke by light gas fired projectiles P. I, II First and second light gas gun stage.

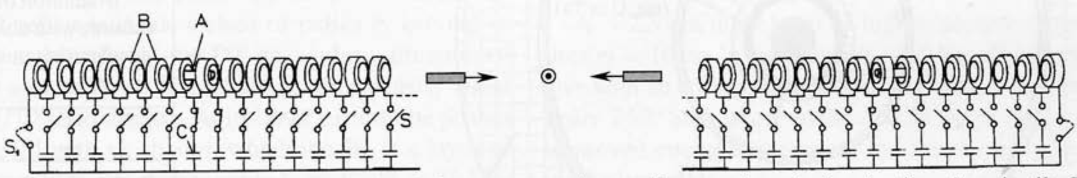

Fig. 4. Bombardment and implosion of a mini-nuke by two magnetic traveling wave launched projectiles. A projectile, B field coils, S_1, S switches.

guns as shown in Figure 3. But because the length of a magnetic traveling wave accelerator to reach a velocity of 20 km/s is by comparison much larger, one may choose to use just two such accelerators as shown in Figure 4.

4. Connection to Laser Fusion by Fast Ignition

A much larger reduction in the critical mass and radial assembly dimension is possible if the implosion is done by powerful laser- or particle beams, where implosion velocities in excess of 100 km/s can be reached. There the fissile core of the mini-nuke assumes the role of a fast ignitor.

In the NIF (National Ignition Facility) at the United States Lawrence Livermore Laboratory, it is planned to compress a DT pellet to $\sim 10^3$ fold solid density with a $\sim$ 100 terawatt-megajoule laser, followed by the ignition in its center by a $\sim$ 150 kJ petawatt laser, where the petawatt laser beam has to drill a hole into the compressed DT fuel. Apart from the uncertainty regarding the feasibility of this fast ignitor concept, the cost of the required $\sim$ 150 kJ petawatt laser is very large. The cost could be substantially reduced by putting a small amount of fissile material in the center of the pellet, replacing the expensive petawatt laser.

Such a configuration would, of course, lead to the release of fission products, but which in this case would be quite small.

5. Fusion-Fission Mini-Nuke Reactor

Even with a yield equivalent to $\sim$ 10 tons of TNT the mini-nuke can still be confined in a chamber of manageable dimensions, and there are no stand-off problems as for electric pulse power driven thermonuclear microexplosions. This is certainly true if the mini-nuke explosion is driven by a high explosive, but also true if the high explosive is replaced by fast moving projectiles.

Comparing the mini-nuke concept as a means for the controlled release of energy by nuclear fusion with magnetic and inertial fusion concepts, we note some advantages and disadvantages.

Some of the advantages are:

1. The concept does not require a large laser or large electric pulse power.

2. With a good fraction of the neutrons absorbed in the combustion products of the high explosive, a much larger fraction of the fusion energy can be directly converted into electric energy by magnetohydrodynamics, letting the fireball expand into a cavity filled with a magnetic field from external field coils.

3. The concept also permits a safe U238 (Th232) fission burn.

The main disadvantages are:

1. The concept depends on the availability of fissile material, which even at the gram size level might work against the overall economy.

2. The release of fission products has the same negative environmental effect as for fission reactors, even though it does not have the run-away (i. e. meltdown) problems of fission reactors.

3. High explosives are not cheap, but a liquid hydrogen-oxygen mixture may be less expensive and be well suited for a high implosion velocity.

Replacing the high explosive with fast moving projectiles almost the same can be said. Instead of being absorbed in the chemical combustion products, a good fraction of the neutrons are absorbed in the projectile mass. For laser- or particle beam induced implosions the presence of a fissile core may facilitate ignition, reducing the demand put on the driver.

6. Mini-Nuke Rocket Propulsion

The reduction of the driver mass through a fissile core in the microexplosion assembly is of importance for nuclear rocket propulsion. There the contamination of space with fission products is of no concern. For space flight research to have any future, a propulsion system is needed which can transport large payloads at high velocities within the solar system. This requires both a high thrust and a high specific impulse. Nuclear thermal propulsion has a high thrust but falls short by about one order of magnitude in the optimal specific impulse. For nuclear electric propulsion the situation is reversed, having a high specific impulse but a small

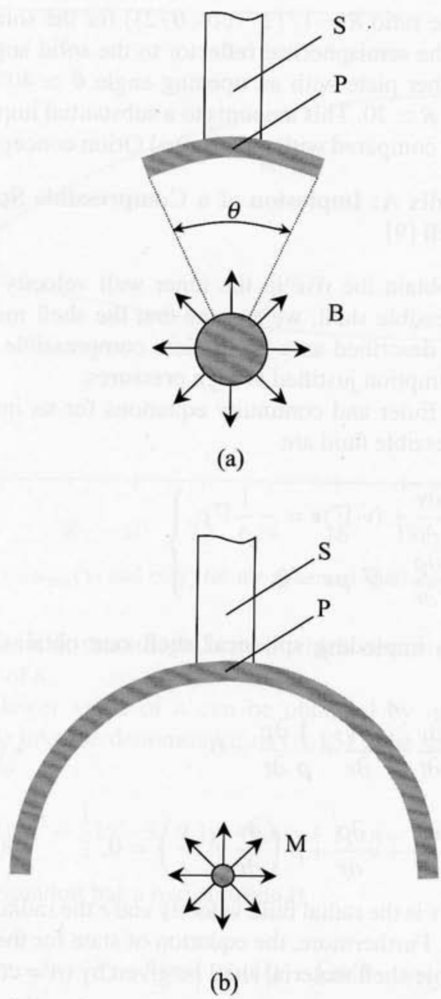

Fig. 5. a) Regular nuclear explosion driven "Orion type" nuclear propulsion system. P pusher, S shaft connecting pusher with space craft, B small fission bomb, θ opening angle to absorb blast from fission bomb. b) The same as in Fig. 5 a), except that here $\theta = 180°$ and B replaced by a mini-nuke M.

thrust. The original Orion-type nuclear bomb propulsion system uses small fission bombs. It has both a high thrust and high specific impulse, but a very low efficiency. There are two reasons for the low efficiency: First, for not too small fission bombs the solid angle for the nuclear explosion to be absorbed by the pusher plate is small, typically with an opening angle $\theta \simeq 10°$. Large opening angles are possible only for small fission explosions with an extravagantly small fission-fuel burn up. It is here where the mini-nukes, because of their small critical mass, can close a gap. Their comparatively small yield permits to replace the pusher plate by a semispherical reflector (see Fig. 5).

With the ratio $R = 1/(1-\cos(\theta/2))$ for the solid angle of the semispherical reflector to the solid angle of the pusher plate with an opening angle $\theta \simeq 40^\circ$, one obtains $R \simeq 20$. This amounts to a substantial improvement if compared with the original Orion concept.

Appendix A: Implosion of a Compressible Spherical Shell [9]

To obtain the rise in the inner wall velocity for a compressible shell, we assume that the shell material can be described as a frictionless compressible fluid, an assumption justified at high pressures.

The Euler and continuity equations for an inviscid compressible fluid are

$$\left.\begin{aligned} &\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p, \\ &\frac{\partial \rho}{\partial t} + \nabla\cdot\rho\mathbf{v} = 0. \end{aligned}\right\} \quad \text{(A.1)}$$

For an imploding spherical shell one obtains from (A.1)

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \quad \text{(A.2a)}$$

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial v}{\partial r} + 2\frac{v}{r}\right) = 0, \quad \text{(A.2b)}$$

where v is the radial fluid velocity and r the radial coordinate. Furthermore, the equation of state for the compressible shell material shall be given by ($A = \text{const}$)

$$p = A\rho^\gamma, \quad \text{(A.3)}$$

hence

$$c^2 = \frac{\mathrm{d}p}{\mathrm{d}\rho} = A\rho^{\gamma-1}, \quad \text{(A.4)}$$

whereby (A.2a-b) can be written as

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\gamma-1}\frac{\partial c^2}{\partial r} = 0, \quad \text{(A.5a)}$$

$$\frac{\partial c^2}{\partial t} + v\frac{\partial c^2}{\partial r} + (\gamma-1)c^2\left(\frac{\partial v}{\partial r} + 2\frac{v}{r}\right) = 0. \quad \text{(A.5b)}$$

To solve these two coupled nonlinear partial differential equations one sets

$$R(t) = (-\alpha t)^n, \alpha = \text{const}, \quad \text{(A.6)}$$

where $R(t)$ is the radius of the inner surface of the collapsing shell as a function of time. For $t < 0$ the radius decreases, reaching $R = 0$ at $t = 0$.

One then introduces the similarity variable

$$\zeta = -\left(\frac{R}{r}\right)^{1/n} = \frac{\alpha t}{r^{1/n}}. \quad \text{(A.7)}$$

Comparison of (A.7) with (A.6) shows that at the inner wall surface $\zeta = 1$, and that for the r-axis $\zeta = 0$.

From (A.6) one obtains for the velocity of the inner wall

$$\dot{R} = -n\alpha R^{1-1/n}. \quad \text{(A.8)}$$

The problem is now reduced to finding the number n, the so-called homology exponent. To obtain it one looks for solutions of the form

$$v = -n\alpha r^{1-1/n}F(\zeta), \quad \text{(A.9a)}$$

$$c^2 = n^2\alpha^2 r^{2-2/n}G(\zeta). \quad \text{(A.9b)}$$

Inserting (A.9a-b) into (A.5a-b), the dependence of r drops out, and one has the two ordinary differential equations

$$\begin{aligned} &(\gamma-1)(1+\zeta F)F' + \zeta G' \\ &\quad + (1-n)((\gamma-1)F^2 + 2G) = 0, \end{aligned} \quad \text{(A.10a)}$$

$$\begin{aligned} &(\gamma-1)\zeta GF' + (1+\zeta F)G' \\ &\quad + [(1-n)(\gamma+1) - 2(\gamma-1)n]FG = 0, \end{aligned} \quad \text{(A.10b)}$$

where $F' \equiv \mathrm{d}F/\mathrm{d}\zeta$, $G' \equiv \mathrm{d}G/\mathrm{d}\zeta$. From (A.8), (A.9a-b) it follows that at the wall, where $\zeta = -1$, one has $F = 1$. Further, for $\zeta = -1$ one must have $G = 0$ with the pressure at the wall surface equal to zero and with it $c^2 = 0$.

With the different set of variables

$$\left.\begin{aligned} x &= \ln(-\zeta), \\ y &= -\zeta F, \\ z &= \zeta^2 G, \end{aligned}\right\} \quad \text{(A.11)}$$

where at the inner wall surface $x = \ln(1) = 0$, (A.10a and b) take the form of three coupled ordinary differential equations

$$\begin{aligned} &\mathrm{d}x : \mathrm{d}y : \mathrm{d}z = [(y-1)^2 - z] \\ &\quad : \left[y(y-1)(ny-1) - 3nyz + \frac{2(1-n)}{\gamma-1}z\right] \end{aligned} \quad \text{(A.12)}$$

Table A.1.

γ	n	m
5/3	0.92	0.087
2	0.835	0.198
3	0.71	0.409
4	0.64	0.563
5	0.60	0.667
6	0.574	0.742
7	0.5574	0.804
8	0.5407	0.8495
9	0.5294	0.8889
10	0.5198	0.9238
11	0.5115	0.9549
12	0.5043	0.9830
13	0.4979	1.0085
14	0.4922	1.0318
15	0.4870	1.0533
16	0.4824	1.0732
17	0.4781	1.0916
18	0.4742	1.1089
19	0.4706	1.125
20	0.4673	1.1402

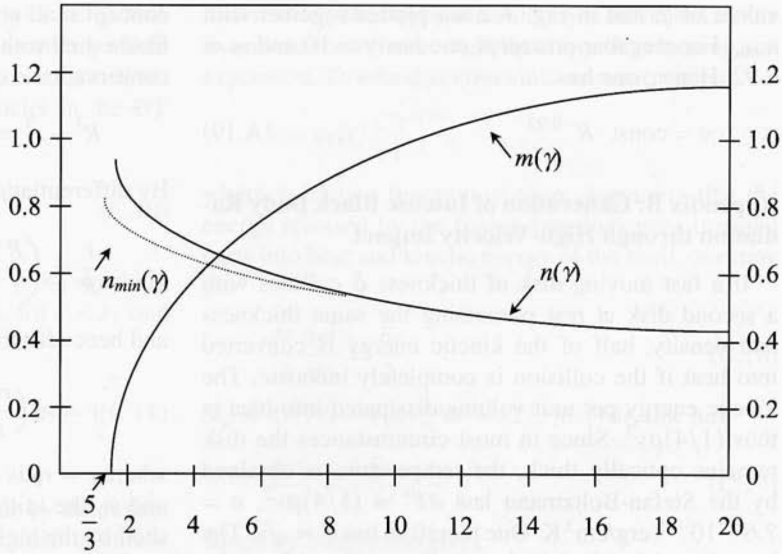

Fig. A.2, The functions $n(\gamma)$, $n_{\min}(\gamma)$ and $m(\gamma)$ for the spherical shell implosion.

$$: \left[2z\left[-nz+n\gamma y^2+\frac{1}{2}y(\gamma-3-(3\gamma-1)n)+1\right]\right].$$

Of these three ordinary differential equations only two are independent. One of them contains two variables only and can be separated from the other two. It is the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}z}=\left[y(y-1)(ny-1)-3nyz+\frac{2(1-n)}{\gamma-1}z\right] \quad (A.13)$$

$$:2z\left[-nz+n\gamma y^2+\frac{1}{2}y(\gamma-3-(3\gamma-1)n)+1\right].$$

With this equation a value of n can be determined from the condition that the solution is regular in passing through a singular point. A differential equation of the form $\mathrm{d}y/\mathrm{d}x=\mathrm{f}(x)/\mathrm{g}(x)$ is singular if both $\mathrm{f}(x)=\mathrm{g}(x)=0$. For the regularity of the solution only one singular point is of importance. In our case it is located on the parabola

$$z=(y-1)^2, \quad (A.14)$$

where according to (A.12) both $\mathrm{d}x/\mathrm{d}y$ and $\mathrm{d}x/\mathrm{d}z$ vanish. On the singular point one has $\mathrm{d}\zeta/\mathrm{d}F=\mathrm{d}\zeta/\mathrm{d}G=0$, which means that F and G are not single valued functions of ζ for an integral curve passing through this point. There is, however, one particular integral curve for a specific value of n where F and G are single valued. There the integral curve in the y-z plane does not have a turning point in crossing the parabola (A.14). It is this integral curve which determines the value of n.

A lower value of n can be obtained by inserting (A.14) into the denominator of (A.13) to be set equal to zero:

$$(\gamma-1)ny^2+\frac{1}{2}(\gamma-3-(3\gamma-5)n)y+1-n=0. \quad (A.15)$$

This equation has a real solution if

$$\left[\frac{1}{2}(\gamma-3-(3\gamma-5)n)\right]^2\geq 4(\gamma-1)n(1-n). \quad (A.16)$$

In the limit where the r.h.s. of (A.16) is set equal to the l.h.s. one obtains a lowest value for n:

$$n_{\min}=\left\{3\gamma^2-6\gamma+7+\left[(3\gamma^2-6\gamma+7)^2-(\gamma-3)^2\cdot(9\gamma^2-14\gamma+9)\right]^{1/2}\right\}\left\{9\gamma^2-14\gamma+9\right\}^{-1}. \quad (A.17)$$

To obtain the exact value of n, the differential equation (A.13) must be integrated numerically from $\zeta=-1$ to $\zeta=0$, that is from $F=1, G=0$, to $F=G=0$, or from $y=1, z=0$ to $y=z=0$.

With $m=1/n-1$ the shell implosion velocity for $R\to 0$ is

$$v=\text{const}\cdot R^{-m}. \quad (A.18)$$

In Table A.1. the values of n and m, obtained by the numerical integration of (A.13), are given for different values of γ, and in Fig. A.2 are plotted together with $n_{\min}$. For megabar pressures one has $\gamma \simeq 10$ and $m = 0.92$. Hence, one has

$$v = \text{const} \cdot R^{-0.92}. \tag{A.19}$$

Appendix B: Generation of Intense Black Body Radiation through High-Velocity Impact

If a fast moving disk of thickness δ collides with a second disk at rest possessing the same thickness and density, half of the kinetic energy is converted into heat if the collision is completely inelastic. The kinetic energy per unit volume dissipated into heat is thus $(1/4)\rho v^2$. Since in most circumstances the disk remains optically thick, the temperature is obtained by the Stefan-Boltzmann law $aT^4 = (1/4)\rho v^2$, $a = 7.67 \cdot 10^{-15}$ erg/cm^3 K. One therefore has $T \propto \sqrt{v}$. The intensity of the black body radiation released from the surface of the hot disk is given by [10]

$$j_\text{r} = -\frac{1}{3\kappa\rho} \nabla(aT^4), \tag{B.1}$$

where

$$\kappa = 7.23 \cdot 10^{24} \rho T^{-3.5} Z^2 / A. \tag{B.2}$$

For aluminum, where $Z^2/A = 6.2$ and $\rho = 2.7$ g/cm^3, one has $(\kappa\rho)^{-1} = 3 \cdot 10^{27} T^{3.5}$ cm. For $v = 50$ km/s one finds that $(1/4)\rho v^2 = 1.7 \cdot 10^{13}$ erg/cm^{2}s$= aT^4$, hence $T = 6.3 \cdot 10^6$ K, and $\kappa\rho = 2 \cdot 10^3$ cm^{-1}. We then find $j_\text{r} \approx 3 \cdot 10^{20}/\delta$ erg/cm^{2}s$= 3 \cdot 10^{13}/\delta a$ W/cm^2. If the disk has a thickness $\delta \simeq 0.3$ cm, one has $j_\text{r} \approx 10^{14}$ W/cm^2, with an energy (1/2) MJ/cm^2 stored in the disk, sufficient to ablatively implode a capsule to a velocity of a few 10^7 cm/s.

Appendix C: Autocatalytic Fission-Fusion Implosions [5]

We consider here a thermonuclear DT target with the tamp made from fissile material. The neutrons released by the thermonuclear reactions cause fission reactions in the fissile shell. If the rate of these reactions is large enough, the fissile shell is heated up to high temperatures, exploding it outwards but also inwards. By its inward implosion it increases the thermonuclear reaction rate in the thermonuclear target with more neutrons released, making more fissions in the shell. It is this coupling of the fission and fusion process which we call an autocatalytic fission-fusion implosion. This concept shall now be analyzed. For an incompressible fissile shell with an outer and inner radius R and r, mass conservation requires

$$R^3 - r^3 = \text{const.} \tag{C.1}$$

By differentiation with regard to time this gives

$$\frac{\dot{r}}{\dot{R}} = \left(\frac{R}{r}\right)^2, \tag{C.2}$$

and hence for the implosion velocity

$$v = v_0 \left(\frac{r_0}{r}\right)^2, \tag{C.3}$$

where $r = r_0$ is the inner shell radius at the time $t = 0$, and v_0 the initial implosion velocity imparted on the shell by the high explosive.

For a compressible shell with an equation of state of the form (p pressure, ρ density, γ specific heat ratio)

$$p = A\rho^\gamma, \quad A = \text{const}, \tag{C.4}$$

the implosion velocity rises less rapidly and is as a function of γ obtained from a gas dynamic similarity solution (see Appendix A). One there has

$$v = v_0 \left(\frac{r_0}{r}\right)^m, \tag{C.5}$$

where $m = m(\gamma)$. For an incompressible shell one has $\gamma \to \infty$ and $m \to 2$. In general $\gamma = \gamma(p)$, with $\gamma \simeq 10$ a typical value for a metallic shell under megabar pressures.

The equation of continuity requires that

$$r^2 \rho v = r_0^2 \rho_0 v_0, \tag{C.6}$$

where ρ_0 is the initial density at $p = 0$. One thus has

$$\frac{\rho}{\rho_0} = \left(\frac{v_0}{v}\right)\left(\frac{r_0}{r}\right)^2, \tag{C.7}$$

or because of (C.5)

$$\frac{\rho}{\rho_0} = \left(\frac{r_0}{r}\right)^{2-m}. \tag{C.8}$$

For $\gamma = 10$ one has $m \approx 1$, hence $\rho/\rho_0 \approx r_0/r$.

The DT reaction releases neutrons at the rate

$$S = \left(\frac{n^2}{4}\right) \langle\sigma v\rangle \left(\frac{4\pi}{3}\right) r^3, \tag{C.9}$$

where n is the DT atomic number density of the plasma and $\langle\sigma v\rangle$ the nuclear reaction cross section particle velocity product, averaged over a Maxwellian. With $N = (4\pi/3)r^3$ the total number of nuclei in the DT plasma, one has

$$S = \left(\frac{N}{4}\right)\langle\sigma v\rangle n. \tag{C.10}$$

The implosion starts from the radius $r = r_0$, reaching ignition at $r = r_1$, where $n = n_1$. Then, for $r < r_1$ one has $n = n_2(r_1/r)^3$ and hence

$$S = S_1\left(\frac{r_1}{r}\right)^3, \tag{C.11}$$

where $S_1 = (N/4)\langle\sigma v\rangle n_1$.

The number of fission reactions made by the fusion neutrons in passing through the fissile shell of thickness δ is

$$f = Sn_f\sigma_f\delta, \tag{C.12}$$

where n_f is the atomic number density of the fissile shell and σ_f the fission cross section. With ε_f, the energy released per fission reaction one has for the total rate of the fission energy in the shell:

$$E_f = f\varepsilon_f = S\delta n_f\sigma_f\varepsilon_f. \tag{C.13}$$

Because of (C.8) one has

$$\frac{n_f}{n_{f1}} = \left(\frac{r_1}{r}\right)^{2-m}, \tag{C.14}$$

where $r_1 < r_0$ is the inner shell radius below which the number of fission reactions becomes important. Because of $\rho\delta^3 = \rho_1\delta_1^3$ one has

$$\delta/\delta_1 = (\rho_1/\rho)^{1/3} = (r/r_1)^{(2-m)/3}, \tag{C.15}$$

and for the rate of the fission energy released in the shell

$$E_f = S_1\left(\frac{r_1}{r}\right)^3\delta_1\left(\frac{r}{r_1}\right)^{(2-m)/3} n_{f1}\left(\frac{r_1}{r}\right)^{2-m} = E_{f1}\left(\frac{r_1}{r}\right)^{\alpha}, \tag{C.16}$$

where

$$E_{f1} = S_1\delta_1 n_{f1}\sigma_f\varepsilon_f, \quad \alpha = (13-2m)/3$$

with $S_1\delta_1 n_{f1}\rho_1$ the respective values for $r = r_1$.

For $r < r_1$ the implosion velocity is increased by the fission reactions in the shell, resulting in its heating and expansion. To take this effect into account, we put

$$v = v_1(t)\left(\frac{r_1}{r}\right)^m, \tag{C.17}$$

where $v_1(t)$ is a function of time. Assuming that the energy released by the fission reactions goes in equal parts into heat and kinetic energy of the shell, one may put

$$\frac{M}{2}\frac{dv_1^2}{dt} = \frac{E_f}{2}. \tag{C.18}$$

Since $dv_1^2/dt = v_1 dv_1^2/dr = (2/3)dv_1^3/dr$, one has

$$\frac{dv_1^3}{dr} = \frac{3}{2}\frac{E_{f1}}{M}\left(\frac{r_1}{r}\right)^{\alpha}, \tag{C.19}$$

which by integration gives

$$v_1^3(t) - v_1^3(r_1) = \frac{(3/2)E_{f1}r_1}{M(\alpha-1)}\left[\left(\frac{r_1}{r}\right)^{\alpha-1} - 1\right] \tag{C.20}$$

with the asymptotic solution

$$v_1 = v_1(0)\left(\frac{r_1}{r}\right)^{(\alpha-1)/3}, \quad r \ll r_1, \tag{C.21}$$

where

$$v_1(0) = \left[\frac{(3/2)(E_{f1}r_1)}{(\alpha-1)M}\right]^{1/3}. \tag{C.22}$$

Inserting (C.21) into (C.17) one obtains

$$v = v_1(r_1)\left(\frac{r_1}{r}\right)^{\beta}, \quad r \ll r_1, \tag{C.23}$$

where

$$\beta = \frac{(10+7m)}{9}. \tag{C.24}$$

One then has

$$\left.\begin{aligned}\frac{n_f}{n_{f1}} &= \left(\frac{r_1}{r}\right)^{2-\beta},\\ \frac{\delta}{\delta_1} &= \left(\frac{r}{r_1}\right)^{(2-\beta)/3};\end{aligned}\right\} \tag{C.25}$$

If, for example, $m = 1$ (corresponding to $\gamma \approx 10$) one finds $\beta \approx 2m = 2$, as if the shell would be incompressible. There one has

$$\left.\begin{aligned}v &= v_1\left(\frac{r_1}{r}\right)^2,\\ n_f &\approx n_{f1},\\ \delta &\approx \delta_1.\end{aligned}\right\} \tag{C.26}$$

In a useful approximation one may match the asymptotic solution (C.21) for $r \ll r_1$, where the fission reactions are taking into account, with the solution (C.5) valid for $r \gg r_1$, above which the fission reactions are small. From this matching condition one can determine a value of $\langle\sigma v\rangle$, above which fusion neutrons become important. With the choice of parameters $n_1 = 5 \cdot 10^{20}$ cm^{-3}, $N_1 = 10^{21}$ one obtains $S_1 = 1.25 \cdot 10^{41}\langle\sigma v\rangle$ s^{-1} and $r_1 = 0.78$ cm. Further, assuming that $\delta_1 \approx 1$ cm, $n_{f1} \approx 10^{23}$ cm^{-3}, $\sigma_f = 2 \cdot 10^{-24}$ cm^2, and $\varepsilon_f = 3 \cdot 10^{-4}$ erg, one finds $E_{f1} = 7.5 \cdot 10^{36}\langle\sigma v\rangle$ erg/s. Assuming that $M = 18$ g (1 cm^3 of U235) and $\alpha = 11/3$ (corresponding to $\gamma \approx 10$), we obtain from (C.22) that $v_1(0) = 5.55 \cdot 10^{11}(\langle\sigma v\rangle)^{1/3}$ cm/s. To match the implosion velocity of 50 km/s with $v_1(0)$ would then require $\langle\sigma v\rangle \simeq 10^{-15}$ cm^3/s, about equal to the optimum value for the DT reaction.

Appendix D: Fission-Fusion Chain Reactions [2]

If fissile material is mixed (homogeneously or inhomogeneously) with neutron-producing thermonuclear material, and if the density and temperature are sufficiently high, thermonuclear fusion reactions releasing neutrons will make fission reactions, raising the temperature of the mixture. Since thermonuclear processes rise with a high power of the temperature in a range where $\langle\sigma v\rangle$ has not yet reached its maximum, the higher temperature will increase the neutron production rate of the thermonuclear material, accelerating the fission reaction rate, and so on. This coupling of the fission and fusion process through the release of heat and rise in temperature shall be called a fission-fusion chain reaction. It effectively increases the neutron multiplication factor reducing the critical mass. This is in particular true at high densities.

To analyze this process we are considering a mixture of fissile (U233, U235, PU239) and fusionable (DT) material. For a given pressure the atomic number densities in the fissile and fusionable material shall be N_U and N_h. Introducing a mixing parameter x, $0 < x < 1$, with $(1-x)N_U$ fissile and xN_h fusionable nuclei per unit volume, the neutron chain reaction in a mixture of infinite extension is determined by the equation [11]

$$\frac{1}{v_0}\frac{\partial\phi}{\partial t} = (\nu - 1)(1-x)N_U\sigma_f\phi + S \tag{D.1}$$

(v_0 velocity of fission neutrons, ν fission neutron multiplication factor, σ_f fission cross section, ϕ neutron flux), where (for DT)

$$S = \frac{1}{4}x^2 N_h^2\langle\sigma v\rangle \tag{D.2}$$

is the source term of the DT fusion reaction neutrons.

We are interested in the temperature range from 1 keV to 10 keV (10^7 K to 10^8 K). There $\langle\sigma v\rangle$ rises rapidly with the temperature dependence (T in keV) [12]

$$\langle\sigma v\rangle \simeq 1.1 \cdot 10^{-20}T^{4.37}. \tag{D.3}$$

With (D.2) and (D.3), (D.1) becomes

$$\frac{1}{v_0}\frac{\partial\phi}{\partial t} = (\nu-1)(1-x)N_U\sigma_f\phi + 2.75 \cdot 10^{-21}x^2N_h^2T^{4.37}. \tag{D.4}$$

Next we need a relation between T and ϕ. As long as $N_e kT > aT^4$ (N_e electron number density, the heat released by the fission and fusion reactions goes mostly into kinetic particle energy. If this inequality is not satisfied the heat goes mostly into black body radiation, and because of the T^4 dependence the temperature rises there only slowly. From the condition $N_e kT > aT^4$, resp. $N_e > (a/k)T^3$ follows that for $T = 10^7$ K (1 keV) $N_e > 5 \cdot 10^{22}$ cm^{-3} and for $T = 10^8$ K (10 keV) $N_e > 5 \cdot 10^{25}$ cm^{-3}. For the intermediate temperature $T = 5 \cdot 10^7$ K only $N_e > 5 \cdot 10^{24}$ cm^{-3} is needed.

The energy released by the fission process per cm^3 and sec is

$$\varepsilon_f(1-x)N_U\sigma_f\phi,$$

where $\varepsilon_f = 180$ MeV $= 2.9 \cdot 10^{-4}$ erg is the fission energy. The energy released in the DT fusion reaction per cm^3 and sec is

$$\varepsilon_\alpha S = \varepsilon_\alpha x^2N_h^2\langle\sigma v\rangle/4 = 2.75 \cdot 10^{-21}\varepsilon_\alpha x^2N_h^2T^{4.37},$$

where $\varepsilon_\alpha = 17.2$ MeV $= 2.75 \cdot 10^{-5}$ erg is the fusion reaction energy. With this heat source the temperature increase in the mixture is

$$3k[g(1-x)N_U + xN_h]\frac{\partial T}{\partial t} = \varepsilon_f(1-x)N_U\sigma_f\phi + 2.75 \cdot 10^{-21}\varepsilon_\alpha x^2N_h^2T^{4.37}, \tag{D.5}$$

where g is the degree of ionization of the fissile material at the temperature T, with $g \approx 10$ a likely value.

Expanding $f(T) = T^{4.37}$ around $T = T_0$ (> 1 keV) into a Taylor series one has

$$T^{4.37} = \text{const} + 4.37 T_0^{3.37} T. \tag{D.6}$$

Inserting (D.6) into (D.4) and (D.5) one obtains

$$\frac{\partial \phi}{\partial t} = \alpha_1 \phi + \beta_1 T + \gamma_1, \tag{D.7}$$

$$\frac{\partial T}{\partial t} = \alpha_2 \phi + \beta_2 T + \gamma_2, \tag{D.8}$$

where

$$\alpha_1 = (\nu - 1)(1 - x) N_{\rm U} \sigma_{\rm f} v_0,$$

$$\beta_1 = 1.2 \cdot 10^{-20} v_0 x^2 N_{\rm h}^2 T_0^{3.37},$$

$$\alpha_2 = \frac{\varepsilon_{\rm f}(1-x) N_{\rm U} \sigma_{\rm f}}{3k[g(1-x)N_{\rm U} + x N_{\rm h}]},$$

$$\beta_2 = \frac{1.2 \cdot 10^{-20} \varepsilon_\alpha x^2 N_{\rm h}^2 T_0^{3.37}}{3k[g(1-x)N_{\rm U} + x N_{\rm h}]}.$$

Furthermore, γ_1 and γ_2 are constants, the values of which are of no interest.

Eliminating ϕ from (D.7) and (D.8) on obtains

$$\ddot{T} - (\alpha_1 + \beta_2)\dot{T} + (\alpha_1 \beta_2 - \alpha_2 \beta_1) T + \alpha_1 \gamma_2 - \alpha_2 \gamma_1 = 0. \tag{D.9}$$

The general solution of (D.9) is the sum of a particular solution of the inhomogeneous equation and the general solution of the homogeneous equation. A particular solution of the inhomogeneous equation is

$$T = \frac{\alpha_1 \gamma_2 - \alpha_2 \gamma_1}{\alpha_2 \beta_1 - \alpha_1 \beta_2} = \text{const}, \tag{D.10}$$

into which the constants γ_1, γ_2 enter, which do not enter into the solution of the homogeneous equation

$$T = \text{const} e^{\lambda t}, \tag{D.11}$$

where

$$\lambda = \frac{\alpha_1 + \beta_2}{2} + \left[\left(\frac{\alpha_1 + \beta_2}{2}\right)^2 + \alpha_2 \beta_1 - \alpha_1 \beta_2\right]^{\frac{1}{2}}. \tag{D.12}$$

For $x = 0$, that is a pure fission assembly one has [9]

$$\lambda = \lambda_0 = (\nu - 1) N_{\rm U} \sigma_{\rm f} v_0. \tag{D.13}$$

If there would be no coupling with the fusion process one would have

$$\lambda_1 = \alpha_1 = \lambda_0 (1 - x). \tag{D.14}$$

If there is a coupling with the fusion process one can define the ratio

$$f = \lambda / \lambda_0 \tag{D.15}$$

from which an effective neutron multiplication factor ν^* can be obtained by putting

$$\nu^* - 1 = (\nu - 1) f. \tag{D.16}$$

Introducing the auxiliary function

$$F(x) = \frac{\dfrac{x^2}{x} \dfrac{N_{\rm h}}{N_{\rm h}}}{g + \dfrac{N_{\rm h}}{1-x} \dfrac{N_{\rm h}}{N_{\rm U}}} \frac{N_{\rm h}^2}{N_{\rm U}^2}, \tag{D.17}$$

one can write

$$\alpha_2 = \frac{2.1 \cdot 10^8 \varepsilon_{\rm f} \sigma_{\rm f} \left(\dfrac{N_{\rm U}}{N_{\rm h}}\right) F(x)}{x^2} \tag{D.18}$$

$$\beta_2 = \frac{2.5 \cdot 10^{-12} \varepsilon_\alpha T_0^{3.37} N_{\rm U} F(x)}{(1-x)}. \tag{D.19}$$

One can see that for $x \sim 0.5$ and $1\,\text{keV} < T < 10\,\text{keV}$ one has

$$\alpha_1 \beta_2 \ll \alpha_2 \beta_1,$$

$$\left(\frac{\alpha_1 + \beta_2}{2}\right)^2 \ll \alpha_2 \beta_1.$$

It therefore follows that approximately

$$\lambda \gtrsim (\alpha_2 \beta_1)^{1/2} \tag{D.20}$$

with the definition (D.15) and with $N_{\rm h}/N_{\rm U} \simeq 43$, (valid at high pressures), one has for $f = f(x)$:

$$f(x) \gtrsim 11.2 \left[\frac{x^2(1-x)}{1+3.3x}\right]^{1/2} T_0^{1.68}. \tag{D.21}$$

This function has a maximum at $x = 0.57$, where

$$f(x) \gtrsim 2.48 T_0^{1.68}. \tag{D.22}$$

According to (D.16), $\nu - 1$ is increased by multiplying it with $f(x)$.