DISPERSION RELATION OF A BOUNDED, MAGNETIZED AND WARM PLASMA *

A.K.T. de Assis and P.H. Sakanaka
Instituto de Física "Gleb Wataghin"
Universidade Estadual de Campinas
13081 - Campinas, SP, Brasil

ABSTRACT

The propagation of the Electromagnetic Waves in a Cylindrical Waveguide filled with uniform magnetized plasma is studied by the warm plasma theory. Dispersion relations are obtained for several situations, such as, zero and finite temperature, and zero, finite and infinite magnetic field. It is found that the waves can not be separated into transverse magnetic and transverse electric modes; only hybrid modes are propagated. For the case of finite magnetic field the Faraday's rotation is obtained.

* Proceedings of the II Latin-American Workshop on Plasma Physics (Medellin, Colombia, 16-28 february, 1987), R. Krikorian (ed.), (World Scientific Publications, Singapore, 1989), pp. 158-184.

I - Introduction

Plasma heating and the generation of non inductive currents by radio frequency waves are very important topics in the Termonuclear Controlled Fusion research. In particular, plasma heating and toroidal current generation in tokamaks by high-power electromagnetic waves may become the definitive solution to the termonuclear controlled fusion problem [1] to [4]. The researchs in the radio-frequency heating of toroidal plasmas have been very great [5] and [6]. When the radio-frequency wavelength is about the tokamak transversal dimensions then it is necessary to make a global treatment of the problem and not a local solution as it is usually done in big tokamaks [7]. Usually the propagation of electromagnetic waves in plasma-filled cylindrical waveguides have been studied according to cold plasma theory [8] to [10]. Wait [11] in 1968 included the electron temperature in a hydromagnetic formulation, which, with Maxwell's equations, gives the compressible plasma theory with which it is possible to explain some facts that the cold plasma theory could not.

The aim of this paper is to apply this compressible plasma theory to the Trivelpiece and Gould problem, [12] and [13], in the case of the plasma temperature be T_o; not using the usual simplifications, as it was done by Ghosh & Pal [14]. The waveguide which limits the plasma is considered to be a perfect conductor of circular cross section and the analysed wave propagations occur along the guide axis. The electroctromagnetic wave and plasma general equations are presented together with the boundary conditions in the plasma conductor interface and the warm plasma dielectric constant is also attained. This is no more the usual scalar matrix as in the cold plasma case or in the infinite plasma case (where the plane wave propagation, [15], is possible) and now it formally rests as a matrix whose elements are partial derivatives which will be applied in the electric field components. In this way it is attained the dispersion relations in several distinct situations that include: null temperature or finite and arbitrary temperature T_o; null external magnetic field, or of value tending to infinity, or of finite and arbitrary value B_o. In the general case ($T_o \neq 0$ and $B_o \neq 0$ but both finite) it is attained a differential equation of the sixth order applied to the electric field longitudinal component. In the literature the results to this case are fourth order differential equations in Ez due to the simplifications which were done in the calculations of these authors [7] and [14]. The other result of this work is that the dispersion relations attained are very general and they include the cases in which the perturbation does not show azimuthal symmetry (that is, the solutions will depend on n, where it is supposed an $e^{-in\theta}$ perturbation).

In section II we will show the basic equations which describe the plasma and their interactions with the electromagnetic wave and the plasma dielectric tensor will be attained too. In section III we will attain the equations for the transversal components of the electric and magnetic fields, $\vec{E}$ and $\vec{H}$, in terms of the longitudinal components, Ez and Hz. In section IV we will attain the equations for Ez and Hz and also the boundary conditions in the plasma-conductor interface. In section V we will show the solutions to these equations and also the dispersion relations in several situations, all of them in a general and explicit form. Throughout the work the International System of Units will be used.

II. The Basic Equations and the Plasma Dielectric Tensor

The Trivelpiece and Gould problem [12] will be studied including now the plasma temperature, as it was derived by wait [11], and the presence of a constant external magnetic field. The plasma is treated as an adiabatic fluid in which the ions are at rest. This approximation is valid in the high-frequency limit, $w \gg w_{ci}$ and $w \gg w_{pi}$, when the ions movement is then completely worthless. In this way the equations which describe the system take the form (equation of continuity, equation of moment transfer, Faraday's equation, Ampère's equation and the equation of state, respectively):

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{u}) = 0 \tag{1}$$

$$nm \left(\frac{\partial}{\partial t} + \vec{u}.\vec{\nabla}\right)\vec{u} = -ne(\vec{E} + \vec{u} \times \vec{B}) - \vec{\nabla}p \tag{2}$$

$$\vec{\nabla} \times \vec{E} = - \mu_o \frac{\partial \vec{H}}{\partial t} \tag{3}$$

$$\nabla \times \vec{H} = - ne\,\vec{u} + \varepsilon_o \frac{\partial \vec{E}}{\partial t} \tag{4}$$

$$p = n\,k_B T = an^{\gamma} \tag{5}$$

where n, $\vec{u}$, m, -e, $\vec{E}$, $\vec{H}$, μ_o, ε_o, p, k_B, T, a, γ are, respectively, the fluid density, the fluid velocity, electron mass, electron charge, electric field, magnetic field, vacuum magnetic permeability, vacuum dielectric constant, plasma pressure, Boltzmann's constant, electron temperature, a proportionality constant and the ratio of specific heats.

To solve these equations a linearization process is used. Considering that in this case there is the presence of a constant external magnetic field $\vec{B}_o$ and observing that in this work the equilibrium situation is stationary, that is, $\vec{u}_o(\vec{r}) = 0$, then the following equations are attained for the first order terms:

$$i\omega p_1 = n_o m\, U^2 \vec{\nabla}.\vec{u}_1 \tag{6}$$

$$i\omega n_o m\vec{u}_1 = n_o e(\vec{E}_1 + \vec{u}_1 x \vec{B}_o) + \vec{\nabla} p_1 \tag{7}$$

$$\nabla x \vec{E}_1 = i\omega\mu_o \vec{H}_1 \tag{8}$$

$$\nabla x \vec{H}_1 = -i\omega\varepsilon_o \vec{E}_1 - n_o e\vec{u}_1 \tag{9}$$

where U is the sound wave velocity in an adiabatic plasma:

$$U = \left(\frac{\gamma\, k_B T_o}{m}\right)^{1/2} \tag{10}$$

These are the basic equation which describe the possible phenomena in this model.

The next step is to obtain the plasma dielectric tensor. From (6), (7) and (9) we obtain:

$$\vec{u}_1 = -\frac{ie}{mw}\left(\vec{E}_1+\vec{u}_1 x\vec{B}_o-\frac{U^2}{w_p^2}\vec{\nabla}(\vec{\nabla}.\vec{E}_1)\right) \tag{11}$$

where w_p is the electron plasma frequency given by:

$$w_p = \left(\frac{n_o e}{m\varepsilon_o}\right)^{1/2} \tag{12}$$

Using the fact that $\vec{B}_o$ is in the direction of the guide wave axis, z axis of the coordinate system, and using (11) and (9) then we obtain

$$\vec{\nabla}x\vec{H}_1 = -iw\,\vec{\vec{\varepsilon}}_t.\vec{E}_1 \tag{13}$$

where

$$(\vec{\vec{\varepsilon}}_t)_{11} = \varepsilon_o\left\{1-\frac{w_p^2}{w^2-w_c^2}+\frac{U^2}{w^2-w_c^2}\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\right.\right.$$

$$\left.\left.-\frac{1}{r^2}-\frac{nw_c}{r^2w}\left(r\frac{d}{dr}+1\right)\right]\right\} \tag{14}$$

$$(\vec{\vec{\varepsilon}}_t)_{12} = \frac{i\varepsilon_o}{w^2-w_c^2}\left\{w_p\quad\frac{w_c}{w}-nU^2\left[\frac{1}{r}\frac{d}{dr}-\frac{1}{r^2}-\frac{nw_c}{r^2w}\right]\right\} \tag{15}$$

$$(\vec{\vec{\varepsilon}}_t)_{13} = \frac{i\varepsilon_o U^2 k}{w^2-w_c^2}\left\{\frac{d}{dr}-\frac{nw_c}{rw}\right\} \tag{16}$$

$$(\vec{\vec{\varepsilon}}_t)_{21} = - \frac{i\varepsilon_o}{w^2-w_c^2} \{ w_p^2 \quad \frac{w_c}{w} - U^2 \, [\frac{w_c}{w} \, (\frac{d^2}{dr^2} +$$

$$+ \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}) - \frac{n}{r^2}(r \frac{d}{dr} + 1)]\} \tag{17}$$

$$(\vec{\vec{\varepsilon}}_t)_{22} = \varepsilon_o\{1 - \frac{w_p^2}{w^2-w_c^2} + \frac{U^2 n}{w^2-w_c^2} \, [\frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} - \frac{n}{r^2}]\} \tag{18}$$

$$(\vec{\vec{\varepsilon}}_t)_{23} = - \frac{\varepsilon_o U^2 k}{w^2-w_c^2} \{\frac{d}{dr} - \frac{n}{r}\} \tag{19}$$

$$(\vec{\vec{\varepsilon}}_t)_{31} = i \; \varepsilon_o U^2 \; \frac{k}{rw^2} \; \{r \frac{d}{dr} + 1\} \tag{20}$$

$$(\vec{\vec{\varepsilon}}_t)_{32} = \frac{\varepsilon_o \; n \; kU^2}{r \; w^2} \tag{21}$$

$$(\vec{\vec{\varepsilon}}_t)_{33} = \varepsilon_o \; \{1 - \frac{w_p^2}{w^2} + \frac{U^2}{w^2} \frac{nk}{r}\} \tag{22}$$

where we supposed a wave propagation in the form $e^{i(kz-n\theta-wt)}$ and where w_c is the electron cyclotron frequency defined by:

$$w_c = \frac{e \; B_o}{m} \tag{23}$$

$\vec{\vec{\varepsilon}}_t$ is then the warm plasm dielectric tensor. It is important to observe that it returns to be the usual cold plasma tensor in the null temperature case (when $T_o = 0$ and $U = 0$). This dielectric tensor will be used to obtain the dispersion relation in the general case. It will be more useful in the case where the wavelength is about the transversal dimensions of the toroid, that is, $\lambda \approx 2R$ (where R is the guide radius). In this case it is no more possible to use the plane-wave approximation and then the spatial derivatives of the electric field components become of fundamental importance.

III. Equations for the Transversal Components of the Electric and Magnetic Fields

Using a standard technique (see, for example, [17] or [14]) we obtain from equations (6) to (9):

$$GEr = nA_1 Ez + nB_1 Hz + C_1 \frac{dEz}{dr} + D_1 \frac{dHz}{dr} \qquad (24)$$

$$iGE_\theta = n \frac{C_1}{r} Ez + n \frac{D_1}{r} Hz + rA_1 \frac{dEz}{dr} + rB_1 \frac{dHz}{dr} \qquad (25)$$

$$GH_r = nA_2 Ez + nB_2 Hz + C_2 \frac{dEz}{dr} + D_2 \frac{dHz}{dr} \qquad (26)$$

$$iGH_\theta = n \frac{C_2}{r} Ez + n \frac{D_2}{r} Hz + rA_2 \frac{dEz}{dr} + rB_2 \frac{dHz}{dr} \qquad (27)$$

$$Ju_r = a_1 E_r + b_1 E_\theta + C_1 Ez + d_1 \frac{dEz}{dr} \qquad (28)$$

$$Ju_\theta = -b_1 Er + a_1 E_\theta + i \frac{w_c}{w} c_1 Ez + i \frac{w_c}{w} d_1 \frac{dEz}{dr} \qquad (29)$$

$$u_z = C_3 Ez \qquad (30)$$

where in these equations we have

$$A_1 = -\frac{ik}{rc^2}\frac{w_c}{w}\left(w_p^2 + \frac{U^2}{x}\frac{k_f^2}{k^2}(\vec{\nabla}_\perp^2 + k_e^2)\right) \tag{31}$$

$$B_1 = \frac{\mu_o w}{r}\left(k_e^2 - \frac{w_c^2}{w^2}k_f^2\right) \tag{32}$$

$$C_1 = ik\left(k_e^2 - \frac{w_c^2}{w^2}k_f^2 + \frac{U^2 k_e^2}{e^2 x k^2}(\vec{\nabla}_\perp^2 + k_e^2)\right) \tag{33}$$

$$D_1 = -\frac{\mu_o w_c w_p^2}{c^2} \tag{34}$$

$$A_2 = -\frac{\varepsilon_o y w}{r}\left(k_e^2 - \frac{w_c^2}{y w^2}k_f^2 + \frac{k_e^2 U^2}{x w^2 y}(\vec{\nabla}_\perp^2 + k_e^2)\right) \tag{35}$$

$$B_2 = -i\,\frac{k w_c w_p^2}{r w_c^2} \tag{36}$$

$$C_2 = -\varepsilon_o w_c\left(k_e^2 - y\ k_f^2 - k_f^2\frac{U^2}{x w^2}(\vec{\nabla}_\perp^2 + k_e^2)\right) \tag{37}$$

$$D_2 = ik\left(k_e^2 - \frac{w_c^2}{w^2}k_f^2\right) \tag{38}$$

$$G = k_e^4 - \frac{w_c^2}{w^2}k_f^4 \tag{39}$$

$$J = I - \frac{w_c^2}{w^2} \tag{40}$$

$$a_1 = -\frac{ie}{mw} \tag{41}$$

$$b_1 = -\frac{e\ w_c}{mw^2} \tag{42}$$

$$c_1 = -\frac{e\, w_c}{mw^2}\,\frac{nU^2}{xrkw_p^2}\,(\vec{\nabla}_\perp^2 + k_e^2) \tag{43}$$

$$d_1 = \frac{e}{mw}\,\frac{U^2}{xkw_p^2}\,(\vec{\nabla}_\perp^2 + k_e^2) \tag{44}$$

$$C_3 = -\frac{ie}{mw}\,(1 - \frac{U^2}{xw^2}\,(\vec{\nabla}_\perp^2 + k_e^2)) \tag{45}$$

$$x = 1 - \frac{U^2}{C^2} \tag{46}$$

$$y = 1 - \frac{w_p^2}{w^2} \tag{47}$$

$$k_e = (\frac{w^2 - w_p^2}{c^2} - k^2)^{1/2} \tag{48}$$

$$k_f = (\frac{w^2}{C^2} - k^2)^{1/2} \tag{49}$$

$$\vec{\nabla}_\perp^2 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{n^2}{r^2} \tag{50}$$

From equations (24) to (50) it is observed that all components Er, E_θ, Hr, H_θ, u_r, u_θ, u_z are obtained only in terms of the longitudinal components of the electric and magnetic fields. This fact reduces the problem of solving equations (6) to (9) to the attainment of Ez(r) and Hz(r). In the next section we will obtain the equations satisfied by these two components, while in the fifth section it will be attained the solutions of these equations and then the dispersion relation.

IV. Equations for the Longitudinal Components of the Electric and Magnetic Fields; Boundary Conditions

From equations (6) to (9) and from manipulations of them (with the rotacional, gradient and divergent operators) we obtain the following equations:

$$(\vec{\nabla}_{\perp}^{2}+k_{e}^{2})Hz = -i\,\frac{\varepsilon_{o}w_{c}}{xk}\,\left[\left(1-\frac{k^{2}U^{2}}{w^{2}}\right)(\vec{\nabla}_{\perp}^{2}+k_{e}^{2}) + xk^{2}\,\frac{w_{p}^{2}}{w^{2}}\right] Ez \tag{51}$$

$$(\vec{\nabla}_{\perp}^{2}+k_{f}^{2})Hz = \frac{-i}{\mu_{o}w_{c}}\,\left[xk\vec{\nabla}_{\perp}^{2} + \frac{1}{xk}\left(\frac{U^{2}}{c^{2}}\,\vec{\nabla}_{\perp}^{2} + k_{e}^{2}\right)\left(\vec{\nabla}_{\perp}^{2} + \frac{w^{2}-w_{p}^{2}}{c^{2}} - \frac{k^{2}U^{2}}{c^{2}}\right)\right] Ez \tag{52}$$

These equations are important because they show that the longitudinal components of the electric and magnetic fields are joined so that the transverse electric, TE, and transverse magnetic, TM, modes can not propagate into the guide. From (51) we obtain the equation for Hz when $B_o = 0$ and also the equation for Ez when $B_o \to \infty$:

$$(\vec{\nabla}_{\perp}^{2} + k_{e}^{2})Hz = 0 \tag{53}$$

$$(\vec{\nabla}_{\perp}^{2} + k_{m}^{2})Ez = 0 \tag{54}$$

where

$$k_m = (k_e^2 + \frac{(c^2-U^2)k^2 w_p^2}{c^2(w^2-k^2U^2)})^{1/2} \quad (55)$$

(51) together with (52) yields:

$$Hz = -\frac{i\epsilon_o U^2 c^2}{xkw_c w_p^2}\{[(\vec{\nabla}_\perp^2 + (\frac{yw^2}{U^2} - \frac{c^2k^2}{U^2})][\vec{\nabla}_\perp^2 +$$

$$+ (\frac{yw^2}{c^2} - \frac{U^2k^2}{c^2})] + xk^2 (\frac{c^2}{U^2} - 1)\vec{\nabla}_\perp^2 - \frac{w_c^2}{U^2}[(1 -$$

$$- \frac{k^2U^2}{w^2})(\vec{\nabla}_\perp^2 + k_e^2) + xk^2 \frac{w_p^2}{w^2}]\} Ez \quad (56)$$

From this equation we obtain the equation for Ez when $B_o = 0$:

$$(\vec{\nabla}_\perp^2+k_e^2)(\vec{\nabla}_\perp^2+k_s^2)Ez = 0 \quad (57)$$

where

$$ks = (\frac{w^2-w_p^2}{U^2} - k^2)^{1/2} \quad (58)$$

Applying the operator $(\vec{\nabla}_\perp^2+k_e^2)$ on both sides of (56) and using (51) it finally yields:

$$(\vec{\nabla}_\perp^6+b_1\ \vec{\nabla}_\perp^4+b_2\ \vec{\nabla}_\perp^2 + b_3)Ez = 0 \quad (59)$$

where

$$b_1 = 2k_e^2 - k_s^2 - \frac{w_c^2}{w^2}\left(\frac{w^2}{U^2} - k^2\right) \tag{60}$$

$$b_2 = k_e^4 + 2k_e^2 k_s^2 - \frac{w_c^2}{w^2}\left[(k_e^2+k_f^2)\left(\frac{w^2}{U^2} - k^2\right) + \right.$$

$$\left. + k^2 w_p^2 \frac{c^2-U^2}{c^2U^2}\right] \tag{61}$$

$$b_3 = k_e^4 k_s^2 - \frac{w_c^2}{w^2} k_f^2 \left[k_e^2\left(\frac{w^2}{U^2} - k^2\right) + k^2 w_p^2 \frac{c^2-U^2}{c^2U^2}\right] \tag{62}$$

Equation (59) together with equation (56) are the fundamental equations for this system and they are valid in the cases in which the temperature and magnetic field have any finite value (though not a null value). This result is more general than that obtained by Ghosh and Pal, [14], because these authors did some simplifications to get the final result and then they attained only a fourth order equation for Ez. The same can be said about the works of Gore and Lashinsky, [7] and of other authors which worked with the cold plasma theory (see Stix, [16]), because then only a fourth order equation for Ez is obtained.

As the conductor is considered to have a null resistivity the boundary conditions are (see Jackson, [17]):

$$Ez(R) = 0 \tag{62}$$

$$E_\theta(R) = 0 \tag{63}$$

$$Hr(R) = 0 \tag{64}$$

$$u_r(R) = 0 \tag{65}$$

where R is the guide radius. As a last boundary condition the dielectric tensor of the plasma is used and the plasma is treated as a dispersive dielectric without free charges, using a treatment similar to that of Trivelpiece, [13]. Then we obtain:

$$(\vec{\vec{\varepsilon}}_t . \vec{E}_1)_r(R) = 0 \tag{66}$$

V. Dispersion Relations

V.1. Case in which $T_o = 0$ and $B_o = 0$

(57) yields, in the limit in which $T_o \to 0$:

$$(w^2 - w_p^2)(\vec{\nabla}_\perp^2 + k_e^2)Ez = 0 \tag{67}$$

The first solution of (67) is $w = w_p$, which does not imply in propagation of energy because the group velocity of this wave is null. The second solution is

$$Ez(r) = A_n J_n(rk_e) \tag{68}$$

where $J_n(x)$ is a Bessel function of n order, and A_n in an arbitrary constant. Applying the boundary condition (62) to this solution, it yields the following dispersion relation:

$$w = \left(\frac{P_{n\gamma}^2}{R^2} c^2 + k^2 c^2 + w_p^2\right)^{1/2} \tag{69}$$

where $p_{n\gamma}$ is a γ order root of the n order Bessel function. The graphic of this dispersion relation is shown in figure 1.**

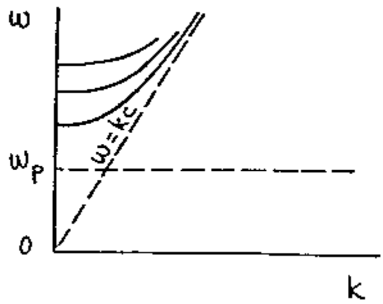

Figure 1: Dispersion Relation when $T_o = 0$ and $B_o = 0$

V.2. Case in which $T_o = 0$ and $B_o \to \infty$

The solution of (65) in the limit in which $T_o \to 0$ is:

$$Ez = A_n J_n(rk_i) \tag{70}$$

where

$$k_i = \left(\frac{w^2-w_p^2}{w^2} \cdot \frac{w^2-k^2c^2}{c^2}\right)^{1/2} \tag{71}$$

Applying the boundary condition (62) it yields two possible solutions:

$$w = \left(\frac{b \pm (b^2-4k^2c^2w_p^2)^{1/2}}{2}\right)^{1/2} \tag{72}$$

where

$$b = k^2c^2 + w_p^2 + \frac{p_{n\gamma}^2 c^2}{R^2} \qquad (73)$$

The graphic of this dispersion relation is shown in figure 2:

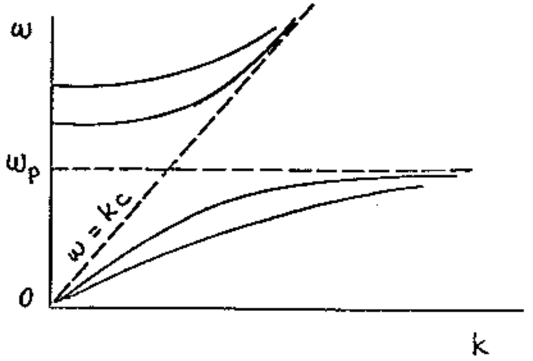

Figure 2: Dispersion Relation when $T_o = 0$ and $B_o \to \infty$

The result (70) is the same that was obtained by Trivelpiece and Gould, [12], in their study of cold plasmas. Here this result appears as a particular case of a more general study.

V.3. Case in which T_o has any Finite Value and $B_o \to \infty$

In this case we have (54) which solution is:

$$Ez = A_n J_n(rk_m) \qquad (74)$$

Applying the boundary condition (62) it yields two possible solutions:

$$w = \left(\frac{f \pm (f^2 - 4k^2c^2(k^2U^2 + w_p^2 + p_{n\gamma}^2 U^2/R^2))^{1/2}}{2}\right)^{1/2} \qquad (75)$$

where

$$f = k^2(c^2+U^2)+w_p^2+\frac{p_{n\gamma}^2\ c^2}{R^2} \qquad (76)$$

The solution w_+ is in the region

$$w > (k^2U^2+w_p^2)^{1/2} \quad ; \quad w > kc \qquad (77)$$

The solution w_- is in the region

$$w < (k^2U^2+w_p^2)^{1/2} \quad ; \quad w < kc \quad ; \quad w > kU \qquad (78)$$

The graphic of thi: dispersion relation is shown in figure 3.

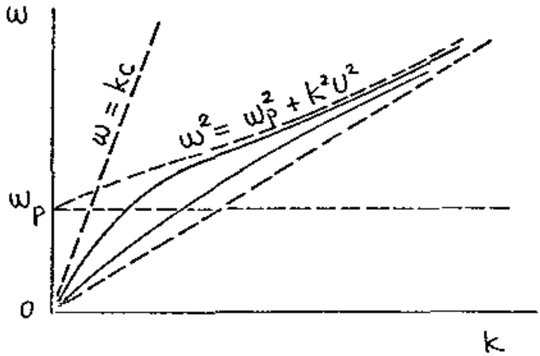

Figure 3: Dispersion Relation when T_o has any Finite Value and $B_o \to \infty$

V.4. Case in which T_o has any Finite Value and $B_o = 0$

The expressions for this case are (53) and (57) and the solutions of these equations are in the form:

$$Ez = A_{1n}J_n(rk_e) + B_{1n}J_n(rk_s) \tag{79}$$

$$Hz = C_{1n}J_n(rk_e) \tag{80}$$

in the region $k_s^2 > 0$ and $k_e^2 > 0$;

$$Ez = A_{2n}I_n(rk_{e_2}) + B_{2n}J_n(rk_s) \tag{81}$$

$$Hz = C_{2n}I_n(rk_{e_2}) \tag{82}$$

in the region $k_s^2 > 0$ and $k_e^2 < 0$; and

$$Ez = A_{3n}I_n(rk_{e_2}) + B_{3n}I_n(rk_{s_2}) \tag{83}$$

$$Hz = C_{3n}I_n(rk_{e_2}) \tag{84}$$

in the region $k_s^2 < 0$ and $k_e^2 < 0$, where

$$k_{e_2} = (k^2 - \frac{w^2-w_p^2}{c^2})^{1/2} \tag{85}$$

$$k_{s_2} = (k^2 - \frac{w^2-w_p^2}{U^2})^{1/2} \tag{86}$$

and where $I_n(x)$ is the modified Bessel function of n order.

Applying (62), (63) and (65) in these solutions we obtain the following dispersion relations:

$$w^2 k_s k_e^3 J_n(Rk_e) J_n'(Rk_s) J_n'(Rk_e) + (k w_p k_e)^2 \cdot$$

$$\cdot J_n(Rk_s) J_n'^2(Rk_e) - \frac{n^2(w^2-w_p^2)w_p^2}{c^2 R^2} J_n(Rk_s) J_n^2(Rk_e) = 0 \qquad (87)$$

in the region $k_s^2 > 0$ and $k_e^2 > 0$;

$$w^2 k_s k_{e2}^3 I_n(Rk_{e2}) J_n'(Rk_s) I_n'(Rk_{e2}) -$$

$$- (k w_p k_{e2})^2 J_n(Rk_s) I_n'^2(Rk_{e2}) +$$

$$+ \frac{n^2(w^2-w_p^2)w_p^2}{c^2 R^2} J_n(Rk_s) I_n^2(Rk_{e2}) = 0 \qquad (88)$$

in the region $k_s^2 > 0$ and $k_e^2 < 0$; and

$$w^2 k_{s2} k_{e2}^3 I_n(Rk_{e2}) I_n'(Rk_{s2}) I_n'(Rk_{e2}) -$$

$$- (k w_p k_{e2})^2 I_n(Rk_{s2}) I_n'^2(Rk_{e2}) +$$

$$+ \frac{n^2(w^2-w_p^2)w_p^2}{c^2 R^2} I_n(Rk_{s2}) I_n^2(Rk_{e2}) = 0 \qquad (89)$$

in the region $k_s^2 < 0$ and $k_e^2 < 0$. In all these equations $J_n'(x)$ and $I_n'(x)$ mean derivative in relation to the argument.

Azakami, Narita and Aye Thein, [18], obtained a dispersion relation for the case $n = 0$ in a similar geometry that coincides with the results (87) to (89) when n is equal to zero. Moreover it is observed that as the dispersion relations present only quadratic terms in n, then the Faraday's rotation is not foreseen in the case in which the warm plasma completelly fills the guide and when $B_o = 0$. The graphic of this dispersion relation is shown in figure 4:

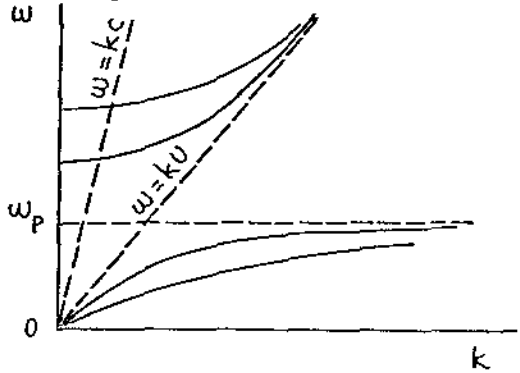

Figure 4: Dispersion Relation when T_o has any Finite Value and when $B_o = 0$

V.5. Case in which $T_o = 0$ and B_o has any Finite Value

The case now analised is that of a cold plasma in which there is the presence of a constant, external and axial magnetic field. In this case Ez is obtained from (59) in the limit in which $T_o \to 0$, while Hz is obtained from (56) in the same limit, which yields:

$$Hz = -\frac{i\varepsilon_o e^2}{kw_c w_p^2}\{(w^2-w_p^2-w_c^2)\vec{\nabla}_\perp^2 + \frac{w^2-w_p^2}{w^2c^2}[w^2(w^2 -$$

$$- w_p^2-k^2c^2-w_c^2) + w_c^2k^2c^2]\}Ez \quad (90)$$

$$(\vec{\nabla}_\perp^2+k_a^2)(\vec{\nabla}_\perp^2+k_b^2)Ez = 0 \quad (91)$$

where

$$k_a^2 = \frac{A-(B)^{1/2}}{D} \tag{92}$$

$$k_b^2 = \frac{A+(B)^{1/2}}{D} \tag{93}$$

$$A = 2w^2(w^2-w_p^2-k^2c^2)(w^2-w_p^2-k^2c^2)-w_c^2w_p^2(w^2+k^2c^2) \tag{94}$$

$$B = w_p^4w_c^2[w_c^2(w^2-k^2c^2)^2+4w^2k^2c^2(w^2-w_p^2)] \tag{95}$$

$$D = 2c^2w^2(w^2-w_p^2-w_c^2) \tag{96}$$

The solution of (91) is, in general (accepting complex arguments):

$$Ez = A_nJ_n(rk_a) + B_nJ_n(rk_b) \tag{97}$$

Using the boundary conditions (62), (63) and (65) we obtain then the following dispersion relation:

$$w^2k^2c^2R[k_aJ_n(Rk_b)J_n'(Rk_a) -$$

$$- k_bJ_n(Rk_a)J_n'(Rk_b)] -nJ_n(Rk_a)J_n(Rk_b) \cdot \frac{B^{1/2}}{w_p^2w_c} = 0 \tag{98}$$

This is a very general result because it is valid for any integer n and it is important to note that in this relation n appear with power one. This indicates that this model foresees the appearance of Faraday's rotation in the case in which the plasma completely fills the guide and when there is the presence of a finite and axial magnetic field. This is due to the fact that the phase velocity of the wave is different in the case in which $n = +N$ and in which $n = -N$. This is a generalization of the Trivelpiece and Gould results, [12], because now the magnetic field $\vec{H}_1$ was considered and then the obtained solution is valid also for fast waves, that is, when the phase velocity is near the light velocity.

V.6. Case in which T_o has any Finite Value and B_o has any Finite Value

In this case the equations that rule the phenomena in all its generality are (56) and (59). Through an algebraic method we can obtain the roots of a cubic equation, as in Smirnov, [19], and it is possible to write (59) as:

$$(\vec{\nabla}_{\perp}^2+k_1^2)(\vec{\nabla}_{\perp}^2+k_2^2)(\vec{\nabla}_{\perp}^2+k_3^2)Ez = 0 \qquad (99)$$

where k_1, k_2 and k_3 are attained in function of the coefficients b_1, b_2 and b_3 of equation (59). The general solution of (99) then rests in the form (accepting complex arguments):

$$Ez = A_n J_n(rk_1)+B_n J_n(rk_2)+C_n J_n(rk_3) \qquad (100)$$

Using the boundary conditions (62), (63), (65) and (66) then we obtain the following dispersion relation for the axially symmetric case (that is, when n = 0):

$$\alpha_{23}J_o(Rk_1)+\alpha_{31}J_o(Rk_2)+\alpha_{12}J_o(Rk_3) = 0 \tag{101}$$

where

$$\begin{aligned}\alpha_{ij} = {} & Rk_ik_j[w_o^2(P_j\eta_i-P_i\eta_j)+U^2Gk(\eta_i+P_i-\eta_j+P_j)] + \\ & + U^2k_ik_j^2\eta_j(\eta_i-P_i)J_o'(Rk_i)J_o''(Rk_j)-U^2k_i^2k_j\eta_i(\eta_j-P_j) \cdot \\ & \cdot J_o''(Rk_i)J_o'(Rk_j)+RU^2k_ik_j\eta_j(\eta_i-P_i) \cdot \\ & \cdot J_o'(Rk_i)J_o'''(Rk_j)-RU^2k_i^3k_j\eta_i(\eta_j-P_j) \cdot \\ & \cdot J_o'''(Rk_i)J_o'(Rk_j)\end{aligned} \tag{102}$$

$$w_o^2 = w^2-w_c^2-w_p^2-\frac{U^2}{R^2} \tag{103}$$

$$P_m = -\frac{GU^2}{\gamma k w_p^2}(k_e^2-k_m^2) \tag{104}$$

$$\eta_m = k(k_e^2 - \frac{w_c^2}{w^2}k_f^2) + \frac{U^2k_e^2}{\gamma k c^2}(k_e^2-k_m^2) + \frac{U^2}{\gamma C^2 k}(a-k_m^2)(b-k_m^2) \tag{105}$$

$$\gamma = \frac{C^2-U^2}{C^2} \tag{106}$$

$$G = k_e^4 - \frac{w_c^2}{w^2}k_f^4 \tag{107}$$

$$a = \frac{s+(s^2-4t)^{1/2}}{2} \tag{108}$$

$$b = \frac{s-(s^2-4t)^{1/2}}{2} \tag{109}$$

$$s = k_e^2 + k_s^2 - \frac{w_c^2}{w^2}\left(\frac{w^2}{U^2} - k^2\right) \tag{110}$$

$$t = \frac{w^2-w_c^2}{w^2}\, k_e^2\left(\frac{w^2}{U^2} - k^2\right) + \frac{k^2 w_p^2}{U^2} \tag{111}$$

(101) is then the general dispersion relation for this model. This result is much more complex than the previous ones, even in this case in which n = 0. In special it is observed that the dispersion relation is obtained from the resolution of a sixth order differential equation, while in the previous work of Ghosh and Pal, [14], they used a fourth order differential equation for Ez. This shows the general treatment of this work.

VI. Conclusion

In this work it was used a fluid plasma model in which only the electrons movement was considered (approximation valid when $w \gg w_{ci}$ and $w \gg w_{pi}$) and then it was applied a linearization process to the adiabatic fluid equations. Then the dielectric tensor of a warm plasma in the presence of a constant external magnetic field was obtained.

Applying the boundary conditions we obtained the dispersion relations in several situations: 1 - Null temperature and null magnetic field; 2 - Null temperature and infinite magnetic field; 3 - Finite temperature and infinite magnetic field; 4 - Finite temperature and null magnetic field (generalization for any n of the results obtained by Azakami, Narita and Aye Thein, [18]);5 - Null temperature and finite magnetic field (where we obtained the Faraday's rotation); 6 - Finite temperature and finite magnetic field, when the dispersion relation is obtained from a sixth order differential equation for Ez. This last case is a generalization of all the previous ones.

This work was supported by FAPESP and CNPq.

** Applying also (63) and (65) to the case of null temperature and null external magnetic field, it can be seen that no propagation is possible. The dispersion relation of this case reduces then to the curve $w = w_p$.

References

[1] Sakanaka, P.H.: "Transport Properties in Tokamaks and Radio-Frequency Heating"; Proc. of the VII Brazilian Symposium on Theoretical Physics, Rio de Janeiro, 1982.

[2] Fisch, N.J.: Phys. Rev. Lett., 41, p. 873, 1978.

[3] Karney, C.F.F. and Fisch, N.J.: Phys. Fluids 22, p. 1817, 1979.

[4] Fisch, N.J. and Boozer, A.H.: Phys. Rev. Lett. 45, p. 720, 1980.

[5] Porkolab, M.: "Radio-Frequency Heating of Magnetic Confined Plasma", FUSION, Vol. 1, Editado por E. Teller, 1981.

[6] Gormezano, C.; Leotta, G.G. and Sindoni, E.: editores: "Heating in Toroidal Plasmas", Proc. of the 3rd Joint Varena-Grenoble International Symposium, 1982.

[7] Gore, R.A. and Lashinsky, H.: Phys. Fluids 22 (11), p. 2718, 1979.

[8] Stix, T.H.: Phys. Rev. 106, p. 1146, 1957.

[9] Dawson, J. and Oberman, C.: Phys. Fluids 2, p. 103, 1959.

[10] Krall, N.A. and Trivelpiece, A.W.: "Principles of Plasma Physics", New York, Plenum Press, 1974.

[11] Wait, J.R.: "Electromagnetics and Plasmas"; Holt, Rinehart and Winston, 1968.

[12] Trivelpiece, A.W. and Gould, R.W.: J. Appl. Phys. 30, p. 1784, 1959.

[13] Trivelpiece, A.W.: "Slow-Wave Propagation in Plasma Waveguides", San Francisco, San Francisco Press, 1967.

[14] Ghosh, S.K. and Pal, S.P.: J. Plasma Physics 29, p. 383, 1983.

[15] Schmidt, H.: "Physics of High Temperature Plasmas", New York, Academic Press, 1979, 2nd edition.

[16] Stix, T.H.: "Theory of Plasma Waves", McGraw-Hill, New York, 1962.

[17] Jackson, J.D.: "Classical Electrodynamics", New York, John Wiley & Sons, 1975, 2nd edition.

[18] Azakami, T., Narita, H. and Aye Thein, U.: Bull. Nagoya Inst. Tech. 24, 201, 1972.

[19] Smirnov, V.I.: "A Course of Higher Mathematics", Vol. I, Pergamon Press, 1964.